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Intermediaries, transport costs
and interlinked transactions

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DISCUSSION PAPER

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**Intermediaries, transport costs
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Mélanie LEFÈVRE¹ and Joe THARAKAN²

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Abstract

Farmers in developing countries often encounter difficulties selling their products on local markets. Inadequate transport infrastructure and large distances between areas of production and consumption mean that farmers find it costly to bring their produce to the market and this very often results in small net margins and poverty amongst farmers who are geographically isolated. Agriculture in developing countries is characterized by the presence of intermediaries that have a transport cost advantage over farmers. Because of their market power, these intermediaries are able to impose interlinked contracts and are free to choose a spatial pricing policy. In this paper, we develop a model of input-output interlinked contracts between a trader and geographically dispersed farmers. We analyze what the welfare implications are as well as the effect on the trader's profit of imposing the use by the trader of either uniform or mill pricing policies, as opposed to spatial discriminatory pricing. We establish under what conditions public authorities can increase farmers' income and reduce poverty in rural areas by restricting the spatial pricing policies that intermediaries can use.

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1 Introduction

NGO's and farmers' organizations in developing countries have been pointing out the negative effects of globalization on farmers in rural areas. They blame unfair foreign competition for pushing farmers out of local markets. Smallholder farming or family farming constitutes about 80 percent of African agriculture. 500 million of such farms provide income to about two-thirds of the 3 billion rural people in the world (FAO, 2008). While a large number of individuals in rural areas in developing countries rely on agriculture, in recent decades small-scale agriculture has suffered; globalization and agro-industrialization cause small farms to go out of business (REARDON and BARRETT, 2000). Small farmers' access to land has been shown to decrease over time (JAYNE et al., 2004). Furthermore, World Bank (2008) reports that an estimated 75% of poor people in developing countries live in rural areas. Most of them depend directly or indirectly on agriculture for their livelihoods. In South Asia and Sub-Saharan Africa, the number of poor people in rural areas is still increasing and is expected to stay above the number of urban poor, at least until 2040 (WORLD BANK, 2008). The prevalence of hunger is still greater in rural than in urban areas (VON BRAUN et al., 2004) and rural children are nearly twice as likely to be underweight as urban ones (UNITED NATIONS, 2010). As on the one hand the agricultural sector in the developing world sees its importance decreasing and on the other hand poverty is increasing, it is important to understand what are the elements which contributed to this situation.

There are different reasons which explain the decline of small-scale agriculture in developing countries. One of these reasons are the high transport costs. Very often urban centers where consumption takes place are close to international transportation routes which gives foreign producers a cost advantage over local producers who face high transport costs. This results in small-scale farmers having difficulties in sell their products on local markets. Another way through which transport costs affect negatively farmers is the cost of access to inputs; this, in turn, reduces their productivity and hence their competitiveness. Inadequate transport infrastructures, combined with large distances between areas of production and areas of consumption, diminish both input use and agricultural production (STAAL et al., 2002, HOLLOWAY et al., 2000). Isolated farmers are less productive (STIFEL and MINTEN, 2008, AHMED and HOSSAIN, 1990, BINSWANGER et al., 1993) and have lower incomes (JACOBY, 2000) and hence face higher poverty than farmers who have an easier access to the market. Small-scale farms, whose income is mainly used to buy food, are especially affected by the importance of transport costs, as they are not able to make the necessary investments to reduce these transport costs.

Evidence suggests that agriculture in developing countries is increasingly characterized by smallholder farmers producing commodities on contract with agro-industrial firms (IFAD, 2003). In Mozambique, 12% of the rural population is working on a contract basis with local enterprises that are affiliated with international companies. In Kenya, 85% of sugar cane production depends on small-scale farmers who provide their production to sugar companies. These intermediaries often possess an advantage over farmers. This advantage can take different forms. For example, it can be the ability to transport the goods at a lower cost (by the use of more efficient transport devices, such as trucks,

or by transforming the product in a way that reduces the volume and/or perishability of the product, etc.). Obtaining this transport cost advantage often requires incurring an important fixed cost, which cannot be borne by a farmer alone. Examples of such intermediaries include maize, beans, roots and tubers in Malawi and Benin (FAFCHAMPS and GABRE-MADHIN, 2006), mandarin in Nepal (POKHREL and THAPA, 2007), cashews in Mozambique (MCMILLAN et al., 2003), etc.

This leads to the question of whether these intermediaries can benefit farmers by helping them market their products, and hence increase their income and reduce poverty amongst them. If this is shown to be the case, then helping setting up these intermediaries (through for example grants or subsidies) would be another way to reduce poverty in rural areas. However, given the characteristics of contracts between intermediaries and farmers, some results in existing theoretical work, e.g. GANGOPADHYAY and SENGUPTA (1987), seem to suggest that farmers would not benefit from the intermediaries' lower costs. The reason is that interactions between intermediaries and farmers involve interlinked transactions. The intermediary not only buys the agricultural output from the farmer, but also provides him with the input that is necessary for his production. With this input-output interlinked contract, the price of both goods are simultaneously fixed. The use of these type of contracts has been documented for various countries and sectors (see for instance WARNING and KEY (2002) for an analysis of the groundnut sector in Senegal; JAYNE et al. (2004) for examples of cash crops production in Kenya; SIMMONS et al. (2005) for an examination of various Indonesian sectors; or KEY and RUNSTEN (1999) for a look at Mexican frozen vegetable industry). While these interlinked contracts have been shown to be efficient, it has also been shown that any efficiency gain is completely appropriated by the trader thereby keeping farmers at their reservation income (e.g. GANGOPADHYAY and SENGUPTA (1987)). Extrapolating this result to our setting, this would mean that because interlinked contracts are used the presence of intermediaries has little effect on the reduction of poverty amongst farmers in rural areas.

However, the result from this literature depends on the assumption that the trader can set a different contract for each farmer. In a spatial context, this corresponds to assuming that the trader can perfectly price discriminate between spatially dispersed farmers. The intermediary collects the product from farmers and sets a different farmgate price. But spatial price discrimination is only one possible pricing policy. There are other modes of collection and hence other pricing policies that intermediaries can choose. While also organizing the collection, an intermediary could pay all farmers the same price, independently of the distance. This corresponds to uniform pricing. Yet another possibility is that farmers are in charge of transport. This corresponds to mill pricing. The choice of a particular pricing policy by the intermediary is important for farmers. Mill pricing, where farmers have to support the transport cost, is disadvantageous for those located far away. Uniform pricing may seem fairer, as all producers receive the same price. However, the closest ones may receive a lower net price than if they were themselves in charge of transport. As both pricing rules are observed in practice, one may ask what drives the choice of a particular pricing policy. A priori, the optimal pricing policy is not obvious, neither from the for-profit intermediary point of view, neither from a social welfare perspective.

In this paper, we analyze interlinked contracts in a spatial context when a trader faces geographically dispersed farmers and is free to choose the spatial pricing policy he uses. As mentioned earlier, we will consider two other spatial pricing policies besides discriminatory pricing. Under uniform pricing, the intermediary is constrained to propose the same contract, hence providing the same income, to all farmers, even if the reservation income is lower for more distant farmers. This implies that, even if such a contract contributes to increasing efficiency, the intermediary is only able to extract part of the efficiency gain. Farmers may gain from the contract and the presence of the intermediary may help to reduce their poverty. Facing the same contract, all farmers will produce the same quantity of the good. Intuitively, in order to induce full participation, the intermediary will propose a contract that gives an income equivalent to the highest reservation income, that is, the income of the farmer who is the closest to the market. The farmers' rents (what they obtain above their reservation income) are increasing with distance, as they all receive the same contract income while reservation incomes decrease with distance.

In the mill pricing case, we have the added complication that not only the farmer's reservation income but also his contract income varies with location. This is because the farmer has to support the transport costs. A consequence of this is that farmers' rents may fail to be monotonic. Indeed, on the one hand, the intermediary wants to encourage farmers to produce efficiently (to generate an efficiency surplus), and, on the other hand, he wants to extract the largest possible share of this generated surplus. As with mill pricing the mill prices are constrained to be the same for all farmers, the intermediary is not able to set the input-output price ratio to its efficient level for each farmer. Only one farmer can be encouraged to produce the efficient quantity of the agricultural good. The farmers located further away underproduce while the ones located closer overproduce. In the presence of nonmonotonic rents, a farmer located in the interior of the market could be pushed down to his reservation level, while others obtain positive rents. Because of the possibility of nonmonotonic rents, unlike most papers in contract theory which rely heavily on the monotonicity of the rent, we cannot use the standard approach to obtain results and have to use an alternative approach to characterize the optimal contract.

In this paper, we show that one of the results of the interlinked contracts literature is more general and still holds under other spatial pricing policies: the intermediary has an interest in providing the input at a price under the market price and also to set a low price for the output. However, when intermediaries are not allowed or unable to discriminate perfectly, farmers may gain from the contract, which implies that the presence of an intermediary may help to reduce their poverty. We establish under what conditions the presence of an intermediary helps to increase farmers' production and income and as well as to reduce poverty, as measured by a Foster-Greer-Thorbecke indicator. We compare the outcomes under different spatial pricing policies (discriminatory, mill and uniform pricing) in terms of income, output of farmers and level of poverty, as well as how the outcome for these different variables varies with a farmer's geographical location. We compare the level of profit an intermediary can obtain under the different spatial pricing policies. This comparison of spatial pricing policies allows us to establish whether a policy recommendation can be made as to the type of spatial pricing policy that should be used by intermediaries. Initially public authorities used to be heavily involved in

the marketing of agricultural products through marketing boards. And very often the pricing policy used by these marketing boards was “pan-territorial pricing” which sets the same price for all farmers irrespective of their geographical location; in other words, this is the equivalent of uniform pricing. State marketing boards used pan-territorial pricing in order to encourage production by poor farmers located in remote areas. Now, most of these marketing boards have disappeared and the intermediaries which have appeared on the market tend to use different types of spatial pricing policies. Policy-makers might want to reduce poverty amongst farmers but be unable to impose a complex tax and subsidy scheme to achieve. We establish whether a restriction on the type of spatial pricing policy that intermediaries are allowed to use could achieve this goal. We determine whether, even though the public authorities are no longer directly involved in the marketing of agricultural products, uniform pricing should be kept as a pricing policy to be used by intermediaries. Some intermediaries are set up with the help of foreign donors with the objective of reducing rural poverty by helping farmers to market their products. Our results establish whether these donors should condition their aid to the use of a particular spatial pricing policy by the intermediary they are helping to set up.

The paper is structured as follows. To illustrate the ideas we develop in our paper, we start in the next section by describing some features of the milk sector in Senegal, which is characterized by the presence of intermediaries who use interlinked transactions and operate in a context where the spatial dimension is important. Section 3 presents the model and its assumptions. Section 4 develops the interlinked transaction model for a for-profit intermediary in the case of spatial price discrimination, which we use as a benchmark. Sections 5 and 6 analyze the cases of uniform pricing and mill pricing respectively. Section 7 discusses the implications of each pricing policy on the trader’s profit, regional differences in farmers’ income, levels of production and poverty amongst farmers. Finally, Section 8 concludes.

2 Characteristics of the milk sector in Senegal

As in most African countries, increased domestic dairy production in Senegal could generate additional income for a large part of the population (STAAL et al., 1997, DELGADO et al., 1999). Indeed, in Senegal 48.12% of the population (73.48% in rural areas) own cattle (ESPS, 2005), most of them being poor: 63.28% of the households involved in agriculture, livestock and forest employment face poverty compared to 37.82% in other employments. In that sense, the development of the dairy sector has the potential to reduce poverty.

Although milk consumption in Africa is still low compared to the rest of the world, dairy products are now part of the consumption habits of most African households. In Senegal, the quantity consumed has quadrupled during the period 1961-1993. Nevertheless, despite this increased consumption, the domestic milk production has risen by less than 40% during the same period, most of the demand being satisfied by an increase in imports (FAOSTAT, 2009).

This stagnation of the domestic milk production is partly due to the characteristics of

the livestock sector: generally, each peasant has only a few cows and each cow provides between 0.5 and 2 liters of milk per day. These two elements result in small quantities, between 2 and 10 liters per day (DUTEURTRE, 2006), of milk being produced. The productivity per animal is determined by its breed (local cattle breeds, Zebu Gobra, Taurine N'Dama or D'jakoré are known to have low productivity) but also by the quantity of animal feed available. About 70% of the Senegalese livestock sector operates in an agro-pastoral system where cattle are raised on pasture but feed supplements are provided by the use of organic manure and harvest residues, in particular from cotton and sesame. One of the main constraints for improving milk production is the difficulty for farmers to obtain these cattle feeds (DIEYE et al., 2005, DIEYE, 2003).

Another factor which hampers the increase of production are high transport costs. The nature of milk makes it difficult to transport it over large distances. While production takes place mainly in rural areas of the country, consumption is concentrated in Dakar, sometimes at more than 300 kilometers from the producers. An inadequate transport infrastructure also contributes to high transport costs.

As incurring large costs for transporting small quantities of milk may turn out to be unprofitable, farmers often prefer not to take part in the market, or to participate only occasionally, resulting in very low quantities of milk being commercialized on the market. In a similar context in Ethiopia, HOLLOWAY et al. (2000) found that each additional minute walk to the collection center reduces the marketable quantity of milk by 0.06 liters per day. In a region where milk yields per day are less than 4 liters, this is of considerable importance. High transport costs also have a negative impact on the use of feed supplements. In Kenya, whose milk sector is comparable to the Senegalese one, STAAL et al. (2002) have found that an additional 10 kilometers between the farmer and Nairobi decreases the probability of using concentrate feed by more than 1%. More isolated farmers are also poorer. In Senegal, while poverty is 35% amongst households who are able to reach a food market in less than 15 minutes, it increases to 63% for those who have to travel more than one hour to reach such a market. This group represents a relatively large share of the population (20%).

Since the Nineties, Senegal as well as other West African countries have seen the emergence of small-scale processing units called "mini-dairies" that play the role of an intermediary between farmers and the market (DIEYE et al., 2005, CORNIAUX et al., 2005). These intermediaries have some kind of advantage over farmers to sell the products on the market. They use more efficient transport devices, such as trucks; they own bulk cooling tanks so that they can stock the milk and do not have to transport it every day to the market, etc. This cost advantage requires a fixed cost, that for isolated farmers with a low income (of which a large part is used to buy food) is important and cannot be borne by each farmer on his own.

These intermediaries seem to expand rapidly in Senegal. Based on a survey conducted in 2002 in Kolda (Southern Senegal), DIEYE et al. (2005) have reported that quantities of milk collected by small-scale processing units in this area increased from 21250 liters in 1996 to 113600 liters in 2001 with the number of processing units increasing from 1 to 5. The quantity collected nearly doubled in the two following years (214205 liters collected in 2003) with the number of intermediaries increasing to 8 (DIEYE, 2006). The same pattern is observed in the other regions (BROUTIN, 2005 and 2008).

Contracts between mini-dairies and farmers often involve interlinked transactions. In the region of Kolda, DIEYE et al. (2005) report that milk processing units provide credit and cattle feed to farmers in order to increase production. The two most important mini-dairies in this region (“Bilaame Puul Debbo” and “Le Fermier”) use three different mechanisms for linking milk purchase and the selling of animal feed: credit for feed purchase, direct feed purchase for the farmer, or guarantee to the feed seller in case of non-payment by the farmer (DIEYE, 2006). In Northern Senegal, “La Laiterie du Berger” buys large quantities of cattle feeds and resells it to the farmers at 50 percent of the market price (BATHILLY, 2007).

The spatial dimension plays a key role in the milk sector. In Senegal, areas of milk production are located far from the capital city (360 km for Richard-Toll where “La Laiterie du Berger” operates, 250 km for Dahra where is the DINFEL collection area), while most of the consumers are located in Dakar. On average, households’ expenditure for milk consumption is 218 CFA per day in Dakar whereas it is 107.5 CFA in other regions (ESPS, 2005). In the rural area, the transport cost is also important compared to the price received by the farmers. In Kolda, where the price received by the producers ranges between 75 and 150 CFA, transport by bicycle costs between 20 and 25 CFA per liter (DIA, 2002). Motorized transport is even more costly; according to one of the managers of “La Laiterie du Berger” (personal interview, 2009), average transport cost on its collection area is 100 CFA per liter, while farmers receive 200 CFA per liter.

To our knowledge, spatial price discrimination is not used in the milk sector in Senegal. Mini-dairies use either uniform or mill pricing. For instance, “La Laiterie du Berger” organizes milk collection and pays all the farmers the same price, independent of the distance. This corresponds to uniform pricing. In “Le Fermier” however, farmers are responsible for transport, such that the ones who are located far from the processing unit receive a considerably lower net price than the closer ones. This corresponds to mill pricing.

In Senegal, the milk production, which stagnated for 30 years, began to increase in the Nineties. One possible explanation for this evolution lies in the emergence of these so-called “mini-dairies.” The theoretical model we develop in the following sections allows us to analyze the impact of the presence of such intermediaries on production, farmers’ income and poverty under different spatial pricing policies when interlinked contracts are used. As explained earlier the presence of interlinked contracts means that farmers will not necessarily gain from contracting with intermediaries. Hence, we cannot immediately conclude from the observation that the production has increased that farmers have effectively gained from this evolution. Our model helps us establish under what conditions poor farmers benefit from the presence of these intermediaries.

3 Model

We analyze the impact of transport costs and interlinked transactions on poverty in the following theoretical framework. A final good market is located at the origin 0 (See Figure 1). We consider one agricultural good which is sold at price p on this market. We assume that the different agents in our model do not have an impact on

this price.¹ This good is consumed at location 0 which can be assumed to be an urban center. Geographical locations are represented along a line. A position x on this line represents a geographical location which is located at a distance x from the market. Furthermore, there is a rural area which starts at a distance r from the urban center and has a geographical extend R . Farmers are uniformly distributed over this rural area.

Figure 1: The model



Each farmer produces the agricultural good according to the same production function $f(k)$, where k is the quantity of input he uses. This input is sold at price i on the market at location 0. The production function has the usual properties: $f(\cdot)$ is twice continuously differentiable, $f(0) = 0$, $f_k = \frac{df}{dk} > 0$, $\lim_{k \rightarrow 0} f_k = \infty$, $\lim_{k \rightarrow \infty} f_k = 0$ and $\frac{d^2f}{dk^2} < 0$. Farmers are assumed to be profit maximizers. A farmer located at x facing farmgate prices $p_F(x)$ and $i_F(x)$ maximizes his income $y(p_F(x), i_F(x))$ by using the optimal quantity $k(p_F(x), i_F(x))$ (for simplicity, as long as it does not cause any confusion, notations $y(x)$ and $k(x)$ will be used):

$$\max_{k(x)} y(x) = p_F(x)f(k(x)) - i_F(x)k(x) \quad (3.1)$$

The existence of an interior solution to this problem is guaranteed by the above assumptions regarding the production function. The choice of input quantity satisfies the following necessary condition:

$$\frac{df}{dk} = \frac{i_F(x)}{p_F(x)} \quad (3.2)$$

As the agricultural good is produced at one location and consumed at another, transport costs have to be incurred to bring this good to the market. These costs are assumed to be linear in distance. To simplify the analysis we assume that transport costs are negligible for the input and set them equal to zero.² A farmer located at a distance x from the market faces a transport cost $t(x) = \tau x$ and hence this farmer can obtain a net per unit price $p_F(x) = p - \tau x$ for the good he produces. In this paper, we analyse how the presence of an intermediary can improve farmers access to the market. We do

¹This can be the case for example because we are in a small open economy and the price of this good is determined on world markets.

²This also reflects the fact that in reality the transport cost for the input is effectively zero as the input is purchased when the output is delivered.

not analyze a related question of whether the presence of an intermediary influences the participation of farmers. Hence we make the following assumption:

Assumption 1. *All farmers are able to profitably sell on the same market as the trader.*

This implies the following restriction on the parameter values, $p > \underline{p} \equiv \tau r + \tau R$.

An intermediary is located at r .³ The intermediary is assumed to have a cost advantage over the farmers. Here, we assume that the trader has an advantage to transport the good between r and 0. Transport costs for the trader are given by $t(x) = \theta r + \tau(x - r)$ per unit of output transported, with $\theta < \tau$. This trader offers contracts to the geographically dispersed producers. Our objective is to analyze how the presence of an intermediary allows farmers to benefit in terms of a better accessibility to markets; we do not analyze the issue of how it affects the participation of farmers. Hence we also make the following assumption:

Assumption 2. *Independently of the pricing policy the trader finds it in its interest to offer contracts to all farmers.*

This implies that we restrict ourselves to certain parameter values.⁴ As in the examples mentioned in Section 2, we consider situations in which a single trader with a cost advantage buys the agricultural good from farmers and sells them an input. Hence, uncertainty does not play a role and there are no incentive problems. If in addition the trader would not be able to enforce nonlinear contracts because he would not be able to prevent arbitrage between agents, the best strategy for the trader is to offer linear interlinked contracts to the farmers (see e.g. Ray (1998), Bardhan and Udry (1999)).⁵ Hence, there is an input-output interlinked relationship between them: on the one hand

³In developing countries, poor infrastructures in rural areas reduce the incentives for intermediaries to locate within these rural areas. By locating just outside of a rural area, the intermediary has a better access to roads to urban centers, electricity, water, etc. Because of the limited number of farmers involved and the potentially large fixed investment costs, further entry would unlikely be profitable. Hence the intermediary is assumed to have monopoly/monopsony power when he trades with the farmers. On the final market, however, the intermediary is price-taker.

⁴The limit $r + R$ can be seen has a physical limit of the production area. It can be due to the existence of a national border, to the absence of farmers beyond a certain distance, or to technical limits for transporting perishable goods over long distances. It can be shown that for parameter values that respect the condition $p > \text{Max}\{\underline{p}, p_R\}$ with $p_R \equiv \frac{(\tau r - \theta r - \tau(R/2))^2}{2(\tau r - \theta r - \tau R)} + \theta r + \tau(R/2)$, the trader finds it optimal to offer contracts to all farmers independently of the pricing policy used (see Appendix F). Alternatively, complete market coverage could be explained not by the fact that it is profitable, but because of social reasons, the trader may not be able to contract only with some farmers of a local community.

⁵The literature has identified different reasons for the emergence of interlinked transactions. Among the different reasons we have rationed or imperfect rural credit (GANGOPADHYAY and SENGUPTA, 1987; CHAKRABARTY and CHAUDHURI, 2001), output market price uncertainty (CHAUDHURI and GUPTA, 1995), risk aversion (BASU, 1983; BASU et al., 2000), unobservable tenant effort (BRAVERMAN and STIGLITZ, 1982; MITRA, 1983) or the inability to collude (MOTIRAM and ROBINSON, 2010).

the trader buys the output from the farmers and, on the other hand, sells them an input necessary for their production. Prices for both input and output are simultaneously fixed in the contract between the trader and the farmer. The trader sells the agricultural output from the farmers and buys input for them on the market located in 0, at market price p and i respectively.

The sequence is the following. In a first step, the trader proposes a contract $(p_C(x), i_C(x))$ to each farmer located on the segment $[r, r + R]$. Very often, the quantities produced by each individual farmer are small. We assume that contract prices do not depend on the quantity sold. The farmer located at x receives $p_C(x)$ per unit of output and pays $i_C(x)$ per unit of input. Each farmer can individually accept or reject the contract. In a second step, each farmer chooses his optimal quantity of input, which determines his level of production. If he has accepted the contract, he faces prices $(p_C(x), i_C(x))$ and chooses optimal input use $k^*(x) = k(p_C(x), i_C(x))$. If he rejects the contract, he sells his production directly to the final market. The same applies to the purchase of inputs. In this case, he chooses the optimal amount of input k^0 , which is a function of market prices (p, i) as well as of the transport cost he has to support, that is $k^0(x) = k(p - \tau x, i)$. In a last step, output is produced and sold on the market, either directly by the farmer (if he has rejected the contract) or via the trader (if the farmer has accepted the contract).

This means that the trader's problem can be characterized as follows:

$$\max_{p_C(x), i_C(x)} \Pi = \int_r^{r+R} [(p - \theta r - \tau(x - r) - p_C(x))f(k^*(x)) + (i_C(x) - i)k^*(x)]dx - F \quad (3.3)$$

where F is the fixed cost necessary to obtain the transport cost advantage,⁶ subject to the demand for input (3.2) and the following participation constraint:

$$y(x) \equiv p_C(x)f(k^*(x)) - i_C(x)k^*(x) \geq y^0(x) \equiv (p - \tau x)f(k^0(x)) - ik^0(x) \quad (3.4)$$

for all $x \in [r, r + R]$. One of the questions we will be looking at is whether, without state intervention, the different outcomes are socially optimal. The efficient input use, $k^\#(x)$, maximizes the sum of trader's profit and farmer's incomes $\int_r^{r+R} (p - \theta r - \tau(x - r))f(k(x)) - ik(x)dx$ and satisfies

$$\frac{df}{dk} = \frac{i}{p - \theta r - \tau(x - r)} \quad (3.5)$$

Given $\theta < \tau$ and the concavity of production function, this implies that for all x , $k^0(x) < k^\#(x)$: in the stand-alone situation, farmers use too little input compared to what is socially optimal.

In the following sections we will look at different ways in which the trader can set contracts with farmers who are geographically dispersed.

⁶Hereafter, we omit this cost F as it has no influence on the optimization result. We assume that F is not too high with respect to the profit that can be made by the intermediary while being too high for a single farmer to incur.

4 Spatial price discrimination

The trader proposes to each farmer a contract $(p_D(x), i_D(x))$ in function of the farmer's location x . Depending on the location of the farmer, this contract can be different and the difference in two farmers' contracts does not necessarily represent the difference in transport costs between them. Each farmer can individually accept or refuse the contract proposed. Hence, to maximize his total profit, the trader chooses a contract which maximizes the profit he makes at each location.

From equations (3.3) and (3.4), the trader's problem may be written as:

$$\max_{p_D(x), i_D(x)} \pi(x) = (p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x) - (p_D(x)f(k^*(x)) - i_D(x)k^*(x)) \quad (4.1)$$

$$\text{s.t. } g(x) \equiv p_D(x)f(k^*(x)) - i_D(x)k^*(x) - y^0(x) \geq 0 \quad (4.2)$$

The Lagrangian is given by:

$$\mathcal{L} = (p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x) + (\lambda(x) - 1)(p_D(x)f(k^*(x)) - i_D(x)k^*(x)) - \lambda(x)y^0(x) \quad (4.3)$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_D(x)}{p_D(x)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = \left((p - \theta r - \tau(x - r)) \frac{i_D(x)}{p_D(x)} - i \right) \frac{\partial k^*(x)}{\partial p_D(x)} + (\lambda(x) - 1)f(k^*(x)) = 0 \quad (4.4)$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = \left((p - \theta r - \tau(x - r)) \frac{i_D(x)}{p_D(x)} - i \right) \frac{\partial k^*(x)}{\partial i_D(x)} + (\lambda(x) - 1)(-k^*(x)) = 0 \quad (4.5)$$

$$\lambda(x) \geq 0, \quad g(x) \geq 0, \quad \lambda(x)g(x) = 0 \quad (4.6)$$

Multiplying (4.4) by $p_D(x)$ and (4.5) by $i_D(x)$, adding these two expressions up, and because input demand $k^*(x)$ is homogeneous of degree zero in both prices, this yields:

$$(\lambda(x) - 1)(p_D(x)f(k^*(x)) - i_D(x)k^*(x)) = 0 \quad (4.7)$$

If the second term were equal to zero, this would imply that $y(x) = 0$ so that $g(x) < 0$, which contradicts (4.6). Thus, the first term has to be equal to zero, which implies that $\lambda(x) = 1$. Substituting this in either of the first order conditions yields:

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p - \theta r - \tau(x - r)} \quad (4.8)$$

Equation (4.8) characterizes the optimal contract $(p_D(x), i_D(x))$. This contract induces the farmer to increase his level of input (as well as his level of output) with respect to the levels he would have chosen in the stand-alone case, even though he receives the same income, as it is stated in the following proposition.

Proposition 1. *Under spatial price discrimination, the trader induces each farmer to use the efficient quantity of inputs, which is larger than in his stand-alone situation ($k^*(x) = k^\#(x) > k^0(x)$), while keeping the farmer at his reservation income level ($y(x) = y^0(x)$).*

Proof of Proposition 1: As the ratio of input price to output price is given by (4.8), this tells us, by using (3.2) and comparing it to (3.5), that the farmer will choose the efficient level of input: $k^*(x) = k^\#(x)$. Given that $\tau > \theta$ and that $f(k)$ is strictly concave and using (3.2) with respectively $(p_F(x), i_F(x)) = (p_D(x), i_D(x))$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we have that $k^*(x) > k^0(x)$. Since $\lambda(x) = 1$, we have from (4.6) that this implies that the individual rationality constraint is binding: $g(x) \equiv p_D(x)f(k^*(x)) - i_D(x)k^*(x) - y^0(x) = 0$. \square

Substituting (4.8) in the binding participation constraint $g(x) = 0$ gives:

$$p_D(x) = (p - \theta r - \tau(x - r)) \underbrace{\frac{(p - \tau x)f(k^0(x)) - ik^0(x)}{(p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x)}}_{\equiv \eta_D(x)} \quad (4.9)$$

$$i_D(x) = i \underbrace{\frac{(p - \tau x)f(k^0(x)) - ik^0(x)}{(p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x)}}_{\equiv \delta_D(x)} \quad (4.10)$$

Note that, with these prices, arbitrage between farmers is impossible: it can be shown that, for any farmer's location x , he has no interest in transporting the good by himself to another location z in order to benefit from the prices $(p_D(z), i_D(z))$. The potential gain from such an action is always lower than the incurred transport cost.

Corollary 1. *Under spatial price discrimination, the trader “loses” on the input trading ($i_D(x) < i$) and “gains” on the output trading ($p_D(x) < p - \theta r - \tau(x - r)$).*

Proof of Corollary 1: As $k^*(x) = k^\#(x)$ (Proposition 1), $\eta_D(x) = \delta_D(x)$ may be written as:

$$\eta_D(x) = \delta_D(x) = \frac{\max_k (p - \tau x)f(k) - ik}{\max_k (p - \theta r - \tau(x - r))f(k) - ik}$$

Using the envelop theorem and since by assumption $\theta < \tau$, this implies that $\eta_D(x) = \delta_D(x) < 1$. Using this result with (4.9) and (4.10) this implies that $p_D(x) < p - \theta r - \tau(x - r)$ and $i_D(x) < i$. \square

GANGOPADHYAY and SENGUPTA (1987) obtain similar results. They analyze inter-linked contracts when the input market is characterized by an imperfection, such that the farmer faces a higher input price than the firm. They show that the trader has an interest to “subsidize” the input and “tax” the output, and that this type of contract allows him to appropriate himself all the efficiency gain (i.e. farmers' incomes are pushed down to their reservation income). In our context, the difference between the trader and the farmer lies in the (output) transport costs, and the previous analysis shows that their results remain valid in this context. If the trader did not propose an interlinked contract but only proposed a contract regarding the output price, he would not have been able to push all the farmers' incomes down to their reservation level. Both instruments, output and input prices, are necessary for the trader to capture completely the efficiency

gain. The strategy of “La Laiterie du Berger” that sells cattle feed to farmers at 50% of the market price (personal interview, 2009) is thus consistent with our analysis. In other contexts also, evidence suggests that in interlinked contracts the input is sold at a discount.⁷

It can be easily seen, as it is done in GANGOPADHYAY and SENGUPTA (1987), that if there were no cost difference between the trader and the farmer (i.e. $\tau = \theta$), the optimal contract would be characterized by $\eta_D(x) = \delta_D(x) = 1$, and the role of the trader would be irrelevant. If he has no cost advantage, the trader is not able to organize the production in a more efficient way than farmers do.

Corollary 2. *Under spatial price discrimination, each farmer “gains” on the input trading ($i_D(x) < i$) and “loses” on the output trading ($p_D(x) < p - \tau x$).*

Proof of Corollary 2: From (4.9), $p_D(x) < p - \tau x$ if

$$f(k^0(x)) - \frac{i}{p - \tau x} k^0(x) < f(k^*(x)) - \frac{i}{p - \theta r - \tau(x - r)} k^*(x)$$

From (3.2), (3.5) and Proposition 1, this is equivalent to

$$f(k^0(x)) - \left. \frac{df}{dk} \right|_{k(x)=k^0(x)} k^0(x) < f(k^\#(x)) - \left. \frac{df}{dk} \right|_{k(x)=k^\#(x)} k^\#(x)$$

This is true provided that the production elasticity $\frac{df}{dk} \frac{k}{f(k)}$ is constant or decreasing in k . The result $i_D(x) < i$ follows from Corollary 1. \square

When involved in the interlinked transaction, each farmer receives a price for the output which is lower than the net price he would have received in the stand-alone situation. This “loss” on the output trading is compensated by a “gain” on the input trading, such that, as Proposition 1 states, each farmer obtains an income $y(x)$ from the contract which is exactly equal to his reservation income $y^0(x)$.

The results show that farmers are treated differently depending on their location. On the one hand, farmers located far from the market receive a lower price for their output, but on the other hand they also pay a lower price for input. Moreover, those farmers receive a smaller share of the net price received by the trader on the market for the output and pay a lower part of the input price. Indeed, from (4.9) and (4.10), it can be shown⁸ that $p_D(x)$, $i_D(x)$, and $\eta_D(x) = \delta_D(x)$ are decreasing in x . Contract

⁷In Kenya, British American Tobacco Ltd delivers input to farmers at prices that are “*in most cases lower than the Nairobi wholesale prices for similar products*”, while Kenya Tea Development Agency Ltd supplies bags of fertilizer at a price “*significantly lower than the wholesale price in Nairobi and much lower than the retail price offered to the smallholders by the village-level stockists*” (IFAD, 2003). Sometimes, input is even given for free (Koo et al., 2012, IFAD, 2003).

⁸The first derivative of $\eta_D(x)$ with respect to x is negative if $f(k^*(x))[(p - \tau x) f(k^0(x)) - i k^0(x)] < f(k^0(x))[(p - \theta r - \tau(x - r)) f(k^*(x)) - i k^*(x)]$. As $\theta < \tau$, a sufficient condition for this to be true is that $k^0(x)/f(k^0(x)) > k^*(x)/f(k^*(x))$ which is ensured by the concavity of the production function and the fact that $k^*(x) > k^0(x)$ from Proposition 1. As $\eta_D(x)$ is decreasing in x , it follows that $p_D(x)$ and $i_D(x)$ are also decreasing in x since $\frac{\partial p_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x} (p - \theta r - \tau(x - r)) - \tau \eta_D(x) < 0$ and $\frac{\partial i_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x} i < 0$.

prices $p_D(x)$ and $i_D(x)$ are increasing with the output market price p .⁹ We also have that $\eta_D(x)$ ($= \delta_D(x)$) increase with p which means that trader's mark-up on the output and discount on the input are lower when p is higher. These results seem to indicate, as mentioned before, that the presence of an intermediary or a trader with a cost advantage, would serve efficiency, increase production, but would not directly benefit farmers. This would mean that setting up intermediaries would not be a way to help farmers. However, spatial pricing discrimination is only one possible pricing policy. We now turn to two other pricing policies and show that in these cases, the results are somewhat modified.

5 Uniform pricing

Under uniform pricing policy, the trader is constrained to propose the same contract (p_U, i_U) to all farmers (where p_U and i_U are independent of x). Each farmer can individually accept or refuse the contract proposed.

The trader's problem can be written as:

$$\begin{aligned} \max_{p_U, i_U} \Pi &= \int_r^{r+R} [(p - \theta r - \tau(x - r))f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)] dx \\ \text{s.t. } g(x) &\equiv p_U f(k^*) - i_U k^* - y^0(x) \geq 0 \quad \forall x \end{aligned}$$

Note that k^* is the same for all farmers, independent of their location (see (3.2) where $p_F(x) = p_U$ and $i_F(x) = i_U$ are independent of x). As farmers are distributed on the interval $[r, r+R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r+R]$. The satisfaction of the constraint for the first farmer (located at r) is sufficient to ensure that it is satisfied for all farmers located further (in $x \in]r, r+R]$). Indeed, as k^* is constant for all x and $y^0(x)$ is strictly decreasing in x , $g(x)$ is strictly increasing in x .

Thus, we can replace the continuum of constraints $g(x) \geq 0$ by the unique constraint $g(r) \geq 0$ (see for instance BOLTON and DEWATRIPONT, 2005: 82). The problem is now the following:

$$\begin{aligned} \max_{p_U, i_U} \Pi &= R \left(\left(p - \theta r - \tau \frac{R}{2} \right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*) \right) \\ \text{s.t. } g(r) &\equiv p_U f(k^*) - i_U k^* - y^0(r) \geq 0 \end{aligned}$$

The Lagrangian is given by:

$$\mathcal{L} = R \left(\left(p - \theta r - \tau \frac{R}{2} \right) f(k^*) - ik^* \right) + (\lambda - R) (p_U f(k^*) - i_U k^*) - \lambda y^0(r) \quad (5.1)$$

⁹For instance, STROHM and HOFFLER (2006) have reported that Deepa Industries in Kenya paid a higher price to potatoes producers than originally agreed because the market price had risen.

Noting that at equilibrium $\frac{df}{dk} = \frac{i_U}{p_U}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_U} = R \left(\left(p - \theta r - \tau \frac{R}{2} \right) \frac{i_U}{p_U} - i \right) \frac{\partial k^*}{\partial p_U} + (\lambda - R) f(k^*) = 0 \quad (5.2)$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = R \left(\left(p - \theta r - \tau \frac{R}{2} \right) \frac{i_U}{p_U} - i \right) \frac{\partial k^*}{\partial i_U} + (\lambda - R) (-k^*) = 0 \quad (5.3)$$

$$\lambda \geq 0, \quad g(r) \geq 0, \quad \lambda g(r) = 0 \quad (5.4)$$

Multiplying (5.2) by p_U and (5.3) by i_U , adding these two expressions up and noting that the input demand k^* is homogeneous of degree zero in both prices, we have

$$(\lambda - R)(p_U f(k^*) - i_U k^*) = 0 \quad (5.5)$$

If the last term were equal to zero, this would imply that $y(r) = 0$ such that $g(r) < 0$, which contradicts (5.4). Thus, the first term has to be equal to zero, that is: $\lambda = R$. Plugging this result into either first order condition yields:

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \tau \frac{R}{2}} \quad (5.6)$$

Equation (5.6) characterizes the optimal contract (p_U, i_U) . This contract implies that each farmer receives the same income from the contract as the stand-alone income of the first farmer.

Proposition 2. *Under uniform pricing, if the trader's cost advantage is large enough ($\tau r - \theta r > \tau(R/2)$) the trader induces each farmer to increase his quantity of inputs with respect to the stand-alone situation ($k^*(x) > k^0(x)$) and the trader keeps the closest farmer at his reservation level ($y(r) = y^0(r)$) while the other farmers obtain a positive surplus from the contract. If the trader's cost advantage is too small ($\tau r - \theta r \leq \tau(R/2)$), he is not able to make a positive profit.*

Proof of Proposition 2: Since $\lambda = R$, we have from (5.4) that this implies that the individual rationality constraint is binding: $g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) = 0$. If $\tau r - \theta r > \tau(R/2)$, given that $f(k)$ is strictly concave and using (3.2) with respectively $(p_F(x), i_F(x)) = (p_U, i_U)$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we have that $k^* > k^0(x)$. If $\tau r - \theta r \leq \tau(R/2)$, we have that $k^* \leq k^0(x)$. Given that $g(r) = 0$, the profit is $\Pi = R \left((p - \theta r - \tau \frac{R}{2}) f(k^*) - i k^* - [(p - \tau r) f(k^0(r)) - i k^0(r)] \right)$. From $k^* \leq k^0(r)$ and $\tau r \leq \theta r + \tau(R/2)$, we have that $\Pi \leq 0$. \square

Contrary to the spatial price discrimination case, when the trader is able to operate profitably under uniform pricing, all the farmers except the first one see an increase in their income with respect to their stand-alone situation. Using this policy, “ La Laiterie

du Berger” claims that its presence has allowed to triple the income of the farmers involved (PHITRUST, 2011).¹⁰

Substituting (5.6) in the binding participation constraint $g(r) = 0$ gives:

$$p_U = \left(p - \theta r - \tau \frac{R}{2} \right) \underbrace{\frac{(p - \tau r)f(k^0(r)) - ik^0(r)}{(p - \theta r - \tau \frac{R}{2})f(k^*) - ik^*}}_{\equiv \eta_U} \quad (5.7)$$

$$i_U = i \underbrace{\frac{(p - \tau r)f(k^0(r)) - ik^0(r)}{(p - \theta r - \tau \frac{R}{2})f(k^*) - ik^*}}_{\equiv \delta_U} \quad (5.8)$$

Corollary 3. *Under uniform pricing, when $\tau r - \theta r > \tau(R/2)$, the trader “loses” on the input trading ($i_U < i$) and “gains” on average on the output trading ($p_U < p - \theta r - \tau(R/2)$).*

Proof of Corollary 3: Note, from (5.6) and (3.5), that $k^* = k^\#(r + (R/2))$. Thus, $\eta_U = \delta_U$ may be written as:

$$\eta_U = \delta_U = \frac{\max_k (p - \tau r)f(k) - ik}{\max_k (p - \theta r - \tau(R/2))f(k) - ik}$$

Using the envelop theorem, $\tau r - \theta r > \tau(R/2)$ implies that $\eta_U = \delta_U < 1$. Using this result with (5.7) and (5.8) implies that $p_U < p - \theta r - \tau(R/2)$ and $i_U < i$. \square

Propositions 2 and 3 imply that the trader is able to make a positive profit only if there exists a *sufficient* advantage in transport cost, i.e. $\tau r - \theta r > \tau(R/2)$.¹¹ In this case, he “loses” on the input trading and “gains” on the output trading, as the average net price he receives on the market is higher than the price he pays to each farmer, similarly to what happens in the spatial price discrimination case. However, if his cost advantage is too small, he is not able to profitably induce farmers to organize production in a more efficient way. This result is in contrast with the result obtained under price discrimination, where the trader is able to exploit his cost advantage, even if the advantage is very small.

As it was the case with spatial price discrimination, when the trader’s cost advantage is large enough, contract prices under uniform pricing p_U and i_U are increasing with the output market price p . The same applies for $\eta_U = \delta_U$, which means that farmers receive a higher share of trader’s gain on the output transaction, but pay a higher share of the input price, when p is higher.

¹⁰Higher income due to the contract is also consistent with empirical evidence in other contexts. Indeed, WARNING and KEY (2002) have estimated an increase in gross agricultural income of 207000 CFA for Senegalese peanut producers that have accepted a contract with “arachide de bouche”. Similarly, SIMMONS et al. (2005) have found that the contracts for seed corn in East Java and for broilers in Lombok made significant contributions to farmers’ capital returns.

¹¹This condition is obviously satisfied if the trader chooses optimally the number of farmers he offers a contract to. See Appendix F.

6 Mill pricing

Under a mill pricing policy, the trader pays the same mill price to all farmers. He proposes the same contract (p_M, i_M) to all farmers (where p_M and i_M are independent of x) and farmers have to support the costs of transporting the good to the trader. Thus, the net price for the output received by the farmer at location x is $p_F(x) = p_M - \tau(x - r)$.

From equations (3.3) and (3.4), the trader's problem may be written as:

$$\max_{p_M, i_M} \Pi = \int_r^{r+R} [(p - \theta r - p_M)f(k^*(x)) + (i_M - i)k^*(x)]dx \quad (6.1)$$

$$\text{s.t. } g(x) \equiv (p_M - \tau(x - r))f(k^*(x)) - i_M k^*(x) - y^0(x) \geq 0 \quad \forall x$$

As farmers are distributed on the interval $[r, r + R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r + R]$. Contrary to the uniform pricing case, which constraint(s) will be binding at the optimum is a priori not obvious. Indeed, one cannot determine a priori whether or not the contract income decreases at a faster rate with distance than the reservation income. As JULLIEN (2000) shows, when both reservation and contract utility depend on the agent's type (in our case, his location), it may be the case that the constraint is binding at either end of the interval of agent's type, but it may also be the case that one or several interior agents face binding participation constraints while agents at the "extremes" of the market do not. In the proof of Lemma 1 (Appendix A), we show that, if the production function is homogeneous, the latter does not occur. Indeed, we show that the outcome will be one of the four following cases: (1) the last participation constraint is binding and only the most distant farmer's income is pushed down to the reservation level while the other farmers obtain a positive surplus. This happens if contract prices p_M and i_M are such that the income from the contract decreases less rapidly with distance than the reservation income; (2) the first participation constraint is binding and only the first farmer's income is pushed down to the reservation level while the other farmers obtain a positive surplus. This is possible if contract prices p_M and i_M are such that the income from the contract decreases more rapidly with distance than the reservation income; (3) all constraints are binding and all farmers are pushed down to their reservation income. This is the case if the trader decides to set $p_M = p - \tau r$ and $i_M = i$; (4) no constraint is binding.

Lemma 1. *Under mill pricing, if the production function is homogeneous of degree $h < 1$, $g(r) \geq 0$ and $g(r + R) \geq 0$ are sufficient to ensure that for all x $g(x) \geq 0$.*

Proof of Lemma 1: See Appendix A.

Using Lemma 1, the problem can be written as:

$$\max_{p_M, i_M} \Pi = \int_r^{r+R} [(p - \theta r - p_M)f(k^*(x)) + (i_M - i)k^*(x)]dx$$

$$\text{s.t. } g(r) \equiv p_M f(k^*(r)) - i_M k^*(r) - y^0(r) \geq 0$$

$$\text{and } g(r + R) \equiv (p_M - \tau R)f(k^*(r + R)) - i_M k^*(r + R) - y^0(r + R) \geq 0$$

In Lemma 2, we prove that the case where contract prices are such that only the first farmer's income is pushed down to the reservation level (case (2) above) is dominated by the replication of the stand-alone situation (case (3)). Indeed, in the first case, the trader induces all farmers to decrease their production, compared to their stand-alone level, which is not optimal from the trader's point of view. Hence, if the first farmer's participation constraint is binding at the optimum, this implies that all participation constraints are binding at the optimum and that $i_M = i$ and $p_M = p - \tau r$.

Lemma 2. *Under mill pricing, if the production function is homogeneous of degree $h < 1$ and $g(r) = 0$ at the optimum, this implies that $g(x) = 0$ at the optimum for all x .*

Proof of Lemma 2: See Appendix B.

6.1 Model with a specific production function

In what follows, we use a particular production function to derive some characteristics of the equilibrium.

Assumption 3. $f(k) = 2\sqrt{k}$.

If the participation constraint of the most remote farmer is binding, this implies that $i_M < i$. If it were not the case, the binding participation constraint would imply that $p_M > p - \tau r$, and this, in turn, would not respect the participation constraint for the other farmers. However, the unconstrained equilibrium could be such that $i_M > i$ and $p_M > p - \tau r$. Indeed, a priori one could think that it could be possible to find a contract such that each farmer loses on the input but gains on the output, while no participation constraint is binding. In what follows, we show that the trader has no interest to do so, such that, at the optimum, $i_M \leq i$ always holds.

Proposition 3. *Under mill pricing and Assumption 3, the profit maximizing contract is characterized by $i_M \leq i$, the trader “loses” on the input trading. On the other hand, the trader “gains” on the output trading ($p_M < p - \theta r$).*

Proof of Proposition 3: See Appendix D.

Corollary 4. *Under mill pricing and Assumption 3, except in the case where the stand-alone case is replicated, the profit maximizing interlinked contract implies that each farmer increases the quantity of input he uses, and hence increases his production, compared to his stand-alone alternative.*

Proof of Corollary 4: The participation constraint has to be satisfied for all x . As the production function is homogeneous, this means that $i_M k^*(x) - ik^0(x) \geq 0$ (see also

Appendix A). From Proposition 3, $i_M \leq i$, which implies $k^*(x) \geq k^0(x)$ for the participation constraints to be satisfied. \square

This result of farmers increasing their output (also obtained under discriminatory and uniform pricing) is consistent with what is observed in the milk sector in Senegal. In particular, “La Laiterie du Berger” claims that the feed supplements it provides to the farmers have helped them to increase their production, especially during the dry season (own interview, 2009). This is also observed in other sectors using interlinked contracts.¹²

Using Lemma 2 and Proposition 3, the problem for the trader under mill pricing can be written as:

$$\begin{aligned} \max_{p_M, i_M} \Pi &= \int_r^{r+R} [(p - \theta r - p_M)f(k^*(x)) + (i_M - i)k^*(x)]dx \\ \text{s.t. } g(r+R) &\equiv (p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0 \\ &\text{and } i - i_M \geq 0 \end{aligned}$$

The Lagrangian is given by:

$$\begin{aligned} \mathcal{L} &= \int_r^{r+R} [(p - \theta r - p_M)f(k^*(x)) + (i_M - i)k^*(x)]dx \\ &+ \lambda ((p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R)) + \mu(i - i_M) \end{aligned} \quad (6.2)$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_M}{p_M - \tau(x-r)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_M} = \lambda f(k^*(r+R)) + \int_r^{r+R} \left[\left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial p_M} - f(k^*(x)) \right] dx = 0 \quad (6.3)$$

$$\frac{\partial \mathcal{L}}{\partial i_M} = -\lambda k^*(r+R) - \mu + \int_r^{r+R} \left[\left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial i_M} + k^*(x) \right] dx = 0 \quad (6.4)$$

$$\lambda \geq 0, \quad g(r+R) \geq 0, \quad \lambda g(r+R) = 0 \quad (6.5)$$

$$\mu \geq 0, \quad i - i_M \geq 0, \quad \mu(i - i_M) = 0 \quad (6.6)$$

Contrary to uniform pricing and spatial price discrimination, under mill pricing the optimum is not always constrained. Whether the optimum is constrained or unconstrained depends on the value of the output price p as well as on the importance of the trader’s cost advantage $\tau - \theta$ compared to the size R of the rural market.

¹²In the Indian poultry sector, RAMASWAMI et al. (2006) have found that contract production is more efficient than noncontract one and that the efficiency surplus is largely appropriated by the processor. In Ethiopia, TADESE and GUTTORMSEN (2009) have estimated that producers of haricot bean who are in relational (interlinked) contract supply about 27% more than farmers in spot markets.

Proposition 4. *Under mill pricing and Assumption 3:*

- *If the trader has a large cost advantage ($\tau r - \theta r > \tau R$) and if the output price is large ($p > \bar{p}$ with \bar{p} unique), then the most distant farmer's income is pushed down to his reservation level ($g(r + R) = 0$) while other farmers obtain a positive surplus from the contract. For a lower output price ($p \in [\underline{p}, \bar{p}]$), all farmers, including the last one, obtain a positive surplus from the contract ($g(x) > 0 \forall x$).*
- *If $\tau R/2 < \tau r - \theta r \leq \tau R$, then the most distant farmer's income is pushed down to his reservation level ($g(r + R) = 0$) while other farmers obtain a positive surplus from the contract for all $p > \underline{p}$.*
- *If $\tau R/3 < \tau r - \theta r \leq \tau R/2$ and if the output price is large ($p > \tilde{p}$ with \tilde{p} unique), then all the farmers' incomes (including the income of the last farmer) are pushed down to their reservation level (for all x $g(x) = 0$). This means that the trader simply replicates the stand-alone situation. For a lower output price ($p \in [\underline{p}, \tilde{p}]$), only the most distant farmer's income is pushed down to his reservation level ($g(r + R) = 0$) while the other farmers obtain a positive surplus from the contract.*
- *If the trader has a small cost advantage ($\tau r - \theta r \leq \tau R/3$), then for all $p > \underline{p}$ all the farmers' incomes (including the income of the last farmer) are pushed down to their reservation level and the stand-alone situation is replicated.*

Proof of Proposition 4: See Appendix E.

These results¹³ show that under mill pricing the optimal pricing by the trader is not always to simply charge farmers the prices they face in a stand-alone situation and to make a profit from the transport cost advantage he has. In particular, if his cost advantage is large enough, the trader uses it to introduce a “distortion” in the prices in order to induce farmers to produce more and hence increase his profit even more.

If the trader's transport cost advantage is large and the output price is low, the optimum is unconstrained, meaning that the contract which is optimal from the trader's point of view leads to higher incomes for all farmers compared to their stand-alone situation. This is due to the low level of the stand-alone income which is a consequence of both low output price and high farmer's transport cost. When the output price is larger, this is no longer possible. Indeed, as θ is bounded at 0, the trader's cost advantage cannot be larger than τ , and cannot compensate the increase in the reservation income due to a higher output price. The result that with a sufficient transport cost advantage

¹³The assumption here is that the trader has to cover completely the market. If the trader optimally chooses his market coverage, it can be shown that for $\tau R/3 \leq \tau r - \theta r \leq \tau R$ the condition $p > \underline{p}$ has to be replaced by the condition $p > p^\alpha(R)$ to ensure complete market coverage. See Appendix F.

for a low output price all farmers benefit from contracting with the trader is interesting in a context where agricultural output prices are often driven down by international competition. This means that if international competition drives down prices all farmers benefit in terms of a higher income from the presence of intermediary if mill pricing is used. In contrast, under the two other pricing policies (discrimination or uniform pricing), for any value of p there is always at least one farmer who is pushed down to his reservation income.

On the contrary, if the trader's transport cost advantage is small and if the output price is large, then the contract which would be optimal from the trader's point of view would lead to incomes for the farmers that are lower than their stand-alone incomes. Indeed, the high output price lead to large reservation incomes that cannot be compensated by the trader's cost advantage as it is too small. In this case, the best the trader can do in order for the farmers to accept the contract, is to replicate their stand-alone situation.

7 Poverty and policy implications

As explained before, Senegalese milk production is characterized by the use of small quantities of input (cattle feed) and the production of small quantities of output. Milk producers have low income and most of them can be considered as poor. The empirical literature on various agricultural sectors in developing countries shows that remote farmers use less inputs (STAAL et al., 2002), produce or sell less (HOLLOWAY et al., 2000, STIFEL and MINTEN, 2008) and have a lower income (JACOBY, 2000) than those who are less isolated. Helping them market their products may contribute to reduce rural poverty and boost socio-economic development in rural areas. In this context, we look at measures regarding pricing by intermediaries that could be adopted by policy makers to increase farmers' production, input use and income.

We have shown that, whatever the pricing policy used, the optimal interlinked contract chosen by an intermediary who has a sufficient transport cost advantage induces each farmer to increase the level of input he uses compared to his input use in the stand-alone case and hence to increase his production. However, this does not always result into an increase in the farmers' incomes as the efficiency gain may be completely acquired by the trader. In what follows, we look at what a policy maker who wants to decrease poverty amongst farmers, but is unable to impose a complex tax and subsidy scheme, should impose as a spatial pricing policy to be used by intermediaries. Our analysis also establishes whether foreign donors setting up intermediaries with the aim to help farmers should restrict the spatial pricing policy used by these intermediaries.

As the farmers are geographically dispersed, they will be affected differently by the different pricing policies. We need an indicator which gives us an aggregate measure of poverty. There are potentially different ways to measure this. To measure poverty amongst farmers, we follow FOSTER et al. (1984) and adopt the following poverty indicator:

$$Pov_\alpha = \frac{1}{R} \int_{r+q}^{r+R} \left(\frac{z - y(x)}{z} \right)^\alpha dx \quad (7.1)$$

where $z > 0$ is poverty line (the income shortfall of the farmer located at x is given by $z - y(x)$), $R - q$ is the number of poor farmers (who have an income lower than z) and α can be seen as a measure of poverty aversion, a larger α giving greater emphasis to the poorest farmers. The larger is Pov_α , the higher is the poverty. In order to establish which pricing policy used by the intermediary performs better in reducing poverty with respect the stand-alone situation, we compare the outcomes of the different pricing policies in terms of this poverty indicator.

We also use the squared coefficient of variation as a measure of the inequality amongst the poor (FOSTER et al., 1984):

$$Inequality = \frac{1}{(R - q)} \int_{r+q}^{r+R} \left(\frac{\bar{y} - y(x)}{\bar{y}} \right)^2 dx \quad (7.2)$$

where $\bar{y} = \frac{1}{(R-q)} \int_{r+q}^{r+R} y(x)dx$ is the average income for the poor farmers. This measure of the inequality is associated with Pov_2 in the sense that it is obtained when $R - q$ and \bar{y} are substituted for R and z in the definition (7.1) with $\alpha = 2$. The indicator defined in (7.2) ranges between 0 and 1, being equal to 0 when perfect equality is satisfied.

If discrimination is possible and costless, in a laissez-faire situation, the for-profit trader will choose to discriminate as it leads to the highest profit. In this situation, the efficient optimum is reached. However, no farmer's poverty is reduced, as they all get the same income as in their stand-alone initial situation. While the presence of a trader who has a transport cost advantage is beneficial from an efficiency point of view, it is not from a poverty reduction one.

A policy maker whose aim is to increase farmers' incomes may want to tax the trader's profit in order to redistribute it amongst farmers. However, it is possible that public authorities in developing countries do not have the capacity of doing so. In what follows, we look at what a policy maker can achieve in terms of poverty reduction by restricting the type of spatial pricing policies that intermediaries can use.

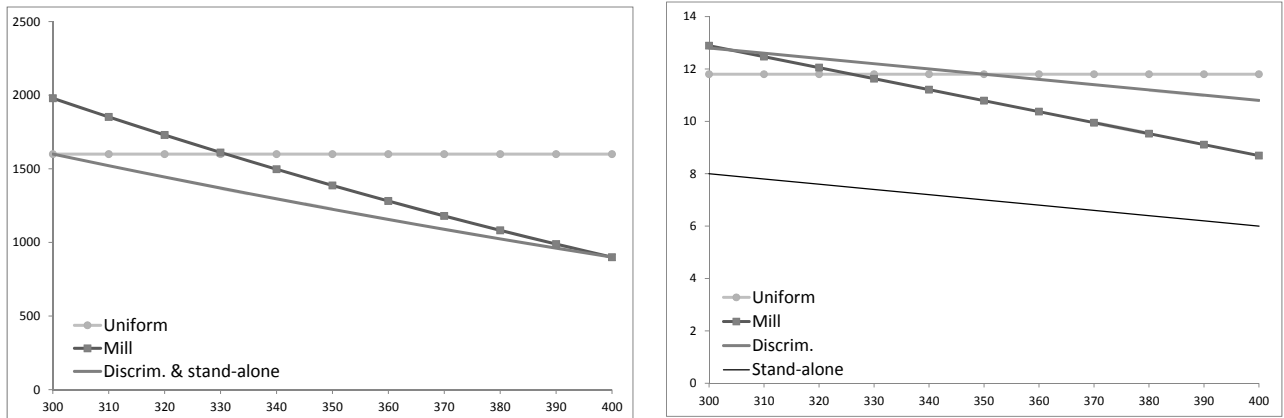
If the trader's transport cost advantage is large enough, imposing uniform pricing leads to an increased income for the poorest farmers, while richer ones are not worse off. Indeed, under this policy, only the farmer the closest to the market, that is, the one who has the highest initial income, is not able to increase his revenue. All the others are able to obtain a positive surplus from the contract, and hence to increase their income. Equality among farmers is ensured, as they all receive the same income and produce the same quantity. However, if the difference in transport cost between the trader and the farmers is small, imposing uniform pricing does not allow the trader to make a positive profit and to exploit his cost advantage to increase production.

If the trader has a sufficiently large cost advantage, requiring him to use mill pricing also increases the income of most farmers. But, contrary to the uniform pricing case, farmers far from the trader, who were already poor, gain less than the one close to the trader. Mill pricing increases inequality amongst farmers, with respect to their stand-alone situation, but also with respect to a situation where the trader is allowed to spatially discriminate.

The previous discussion is illustrated by Figure 2, which represents farmers' income and output as a function of distance, under the three pricing policies when $\tau r - \theta r > \tau R$

and $p > \bar{p}$. Both uniform and mill pricing policies have positive effects on the income of most of farmers. Hence, if the policy maker is concerned only by farmers' income, spatial price discrimination should be prohibited.

Figure 2: Comparison of spatial pricing policies
 (a) Farmer's income $y(x)$ (b) Output produced $f(k(x))$



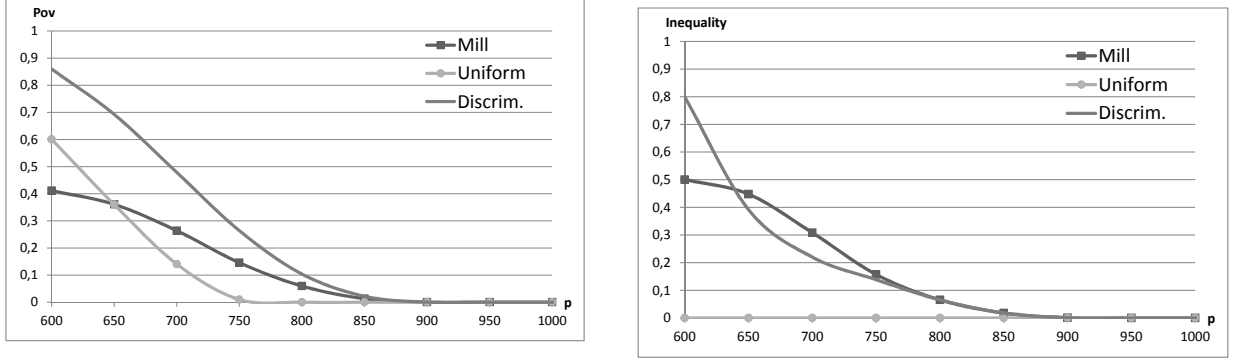
* Choice of the parameters: $r = 300$, $R = 100$, $p = 700$, $\tau = 1$, $i = 100$ and $\theta = 0.2$. The parameters values are such that the uniform pricing contract is profitable for the trader and such that the mill pricing contract is constrained for the last farmer.

Producers' organizations in developing countries and NGOs argue that prices for agricultural goods are too low and claim that they remain low due to "unfair" international competition caused by subsidized exports from industrialized countries. This is seen as one of the reasons which keeps small producers in poverty (see for instance OXFAM (2002) or CFSI (2007) on the milk sector). In a context in which p is very low, imposing mill pricing to a trader who has a large cost advantage may result in increasing all farmers' income, including the most distant one. Numerical simulations also show that, when p is small, mill pricing may be preferred to uniform pricing by a majority of farmers¹⁴ and that the sum of all farmer's incomes may be higher under mill pricing. If the policy-maker's objective is to choose a policy that increases farmers' total income and/or is preferred by the majority of them, then he should impose mill pricing when output price p is low. However, when the output price is high, uniform pricing is preferred by a majority of farmers and leads to a higher total farmers' income, even if the first farmer's income is always pushed down to his reservation level.

Regarding poverty, as measured by the indicator defined in (7.1), spatial price discrimination does not contribute to poverty reduction, as it does not permit to increase farmers' income. Numerical simulations (see Figure 3 (a)) show that mill pricing tends to perform better in reducing poverty for low values of p while uniform pricing dominates when the output price is larger. Note that, when poverty aversion is large, that is α is

¹⁴That is, the median farmer located in $r + R/2$ has a higher income under mill than under uniform pricing.

Figure 3: Comparison of spatial pricing policies
(a) Poverty (b) Inequality



* Stand-alone situation corresponds to spatial price discrimination. Choice of the parameters: $z = 1000$, $r = 300$, $R = 100$, $\tau = 1.5$, $i = 100$ and $\theta = 0.2$ as well as $\alpha = 2$. The parameter values satisfy Assumption 2.

large (not represented here), uniform pricing dominates mill pricing in terms of poverty reduction, as more emphasis is given to the poorest (the most distant farmers) who have a larger income under uniform pricing. With very large α , Pov_α approaches a Rawlsian measure which considers only the income of the poorest farmer. If the policy maker has a Rawlsian objective, the uniform pricing policy should always be encouraged.

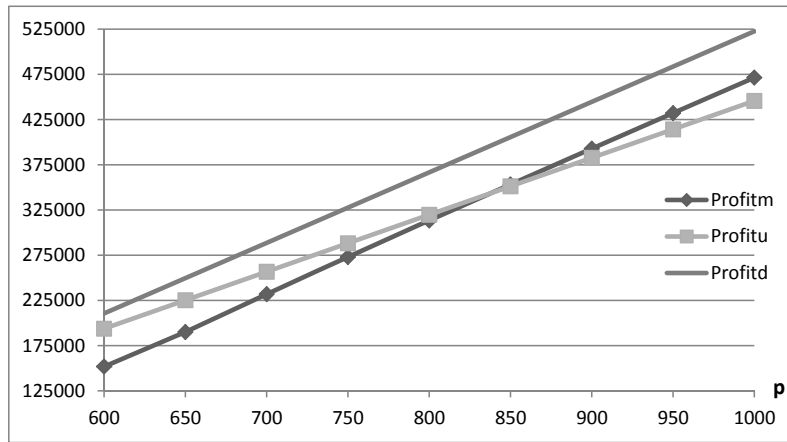
The effect of the pricing policies on the inequality amongst the poor is illustrated in Figure 3 (b). It can be seen that uniform leads to perfect equality, as all farmers get the same income, while mill pricing may lead to the highest level of inequality, the closest farmers being favored with respect to the most distant ones.

The question which remains is whether the intermediary will choose the pricing policy which is optimal from the point of view of poverty reduction or if an intervention by the policy marker is necessary. Figure 4 shows the level of profit for the different pricing policies in function of output price p .

Not surprisingly, for all three spatial pricing policies, the profit is increasing in output price p . Not surprisingly either, price discrimination dominates the two other pricing policies. The ranking between mill pricing and uniform pricing depends on the level of the output price: for a low level of p , the intermediary will prefer uniform pricing while for a high level of p the intermediary will prefer mill pricing. If the intermediary's cost advantage is small, imposing uniform pricing will result in a negative profit. Under mill pricing and discriminatory pricing, however, the trader is able to contract profitably with all farmers, whatever the level of cost advantage.

Putting the information of the last two figures together yields the following conclusions. When p is low, the intermediary prefers uniform pricing while mill pricing leads to the lowest level of poverty. When p is high, we have the opposite result: mill pricing is preferred by the intermediary while it is uniform pricing which is the best in terms of poverty reduction. In these cases, the public authorities should impose a particular pricing policy to the intermediaries if its objective is to reduce poverty amongst geographically dispersed farmers. Only in the case of intermediary values of p will the in-

Figure 4: Comparison of spatial pricing policies in terms of profit



* Choice of the parameters: The parameters values are the same as those for Figure 3.

termediary choose the pricing policy which is also the optimal policy in terms of poverty reduction.

8 Conclusions

In this paper, we develop a model of input-output interlinked contracts between a trader and geographically dispersed farmers, and analyze the implications of different spatial pricing policies used by this trader. We look at three different spatial price policies, namely spatial price discrimination, uniform pricing and mill pricing.

We assume an agricultural output market that is characterized by large transport costs. The intermediary has a (transport) cost advantage over the farmers from whom it buys their production. This cost difference leads to an input-output interlinked contract between the intermediary and the farmer. A first result is that the use of an interlinked contract by a trader who has a sufficient transport cost advantage leads to an increase of the farmer' production, independently of the type of pricing policy used by the intermediary.

If the for-profit intermediary is able to perfectly discriminate contracts between farmers, this would be his preferred option. This allows him to push all the farmers' incomes down to their stand-alone initial income and hence appropriate all the efficiency gain generated by the contract. If this is the case, the presence of the intermediary, while improving agricultural efficiency, does not directly help to reduce rural farmers poverty. In practice discriminatory pricing might not be feasible and other pricing policies exist, such as uniform pricing, where the trader bears the transport costs and concludes the same contract with all the farmers, or mill pricing, where farmers are in charge of transport, and receive the same price at the mill. If the trader's cost advantage is large enough, we show that in both cases, most farmers obtain a positive surplus from the

contract, while the trader is still able to make a profit. In the mill pricing case, under some conditions we can have a situation in which all the farmers, including those located the furthest from the market, see an increase in their income.

We show that imposing a uniform pricing policy to the trader who has a sufficiently large cost advantage leads to an increase of isolated farmers' income. Providing the same income to all farmers, uniform pricing favors relatively more isolated farmers, since they are the ones who initially receive a lower income. Moreover, when the output market price is large enough, uniform pricing also leads to a reduction of farmers' poverty, as measured by a Foster-Greer-Thorbecke indicator. In this case, it is also preferred to mill pricing by a majority of farmers, and it leads to higher total farmers' income.

In developing countries, agricultural market prices are often driven down by international competition. If output market prices are very low, imposing mill pricing may be the best alternative. Indeed, it may increase all farmers' income, including the closest and the most distant one. This is not possible under uniform pricing, whatever the output market price. When the output market price is low, mill pricing performs better in reducing poverty than uniform pricing does. Moreover, there may be cases in which both total farmers' income and median farmer's income are higher under mill than under uniform pricing. Additionally, if the trader only has a small cost advantage, under mill pricing he still may be able to increase most of the farmers' income, while under uniform pricing he cannot profitably contract with the farmers.

We also generalize the result found in GANGOPADHYAY and SENGUPTA (1987) that the trader has an interest in giving a discount to the farmer on the input price. If the trader's cost advantage is sufficiently large, this is true for all three pricing policies considered.

The model developed here gives potential avenues for future research. First, in certain cases, the choice of the size of the collection area may be important to the trader. In that case, rather than considering the number of farmers as being fixed, the number of participants may constitute a choice variable for the trader. A possible extension of our model would consider how the number of suppliers is endogenously chosen. This would also allow to analyze the impact of pricing policy choice on the inclusion of isolated farmers in a collection area.

Appendices

A Proof of Lemma 1

Using the envelop theorem, we have for a participation constraint at location x

$$\frac{\partial g(x, p_M, i_M)}{\partial x} = -\tau (f(k^*(x)) - f(k^0(x))) \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow k^*(x) \begin{matrix} \geq \\ \leq \end{matrix} k^0(x) \quad (8.1)$$

Define \tilde{x} as a location where the participation constraint is binding for a couple (p_M, i_M) , i.e. $g(\tilde{x}, p_M, i_M) = 0$.

If $f(k)$ is homogeneous of degree h , then, using Euler's theorem, the farmer's income is given by $y(x) = i_M k^*(x) \left(\frac{1}{h} - 1\right)$ while his reservation income is given by $y^0(x) = i k^0(x) \left(\frac{1}{h} - 1\right)$. Thus $g(\tilde{x}, p_M, i_M) = (i_M k^*(\tilde{x}) - i k^0(\tilde{x})) \left(\frac{1}{h} - 1\right) = 0$, or equivalently, $k^*(\tilde{x}) = (i/i_M) k^0(\tilde{x})$. Using this result, we can evaluate (8.1) at $x = \tilde{x}$ which yields:

$$\left. \frac{\partial g(x, p_M, i_M)}{\partial x} \right|_{x=\tilde{x}} \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow i_M \begin{matrix} \leq \\ \geq \end{matrix} i \quad (8.2)$$

Together these elements imply that the optimum is characterized by one of the following cases: (1) There is no $\tilde{x} \in [r, r + R]$ implying that $g(x) > 0$ for all $x \in [r, r + R]$, (2) If $i_M < i$, the only possible value for \tilde{x} is $\tilde{x} = r + R$, i.e. $g(r + R) = 0$ and $g(x) > 0$ for all $x \in [r, r + R[$, (3) If $i_M > i$, then the only possible value for \tilde{x} is $\tilde{x} = r$, i.e. $g(r) = 0$ and $g(x) > 0$ for all $x \in]r, r + R]$, and (4) If $i_M = i$, then this means that if the participation constraint is binding somewhere, it has to be binding everywhere: $g(x) = 0$ for all $x \in [r, r + R]$. Hence $g(r) \geq 0$ and $g(r + R) \geq 0$ are sufficient to ensure that $g(x) \geq 0$ for all $x \in [r, r + R]$. \square

B Proof of Lemma 2

Case (2) is characterized by $p_M > p - \tau r$, $i_M > i$ and $g(r) = 0$ as well as $g(x) > 0$ for $x \in]r, r + R]$ at the optimum. The trader's profit can be written as $\Pi^{case2} = \int_r^{r+R} (p - \theta r - \tau(x - r)) f(k^*(x)) - i k^*(x) - y(x) dx$. To have $g(x) > 0$ for $x \in]r, r + R]$, we have to have that $\left. \frac{\partial g(x, p_M, i_M)}{\partial x} \right|_{x=r} > 0$ and, from (8.2), $i_M > i$. From (8.1), this would imply $k^*(r) < k^0(r)$. As the production function is concave, using (3.2), it would imply $\frac{p_M}{i_M} < \frac{p - \tau r}{i}$. Subtracting $\frac{\tau(x-r)}{i}$ on both sides and given that $i_M > i$, this would give $\frac{p_M - \tau(x-r)}{i_M} < \frac{p - \tau x}{i}$, thus $k^*(x) < k^0(x) \forall x$. Compared to Case (2), the trader can always obtain a higher profit by replicating farmers' stand-alone situations (that is, proposing a contract where $p_M = p - \tau r$ and $i_M = i$, such that each farmer uses $k^0(x)$ and obtains his reservation income $y^0(x)$). In this case the profit is given by $\Pi^{case3} = \int_r^{r+R} (p - \theta r - \tau(x - r)) f(k^0(x)) - i k^0(x) - y^0(x) dx$. We have that $\Pi^{case3} > \Pi^{case2}$. Indeed, from the participation constraints, $y(x) \geq y^0(x)$, and, given our assumptions on $f(k)$, the function $(p - \theta r - \tau(x - r)) f(k(x)) - i k(x)$ is concave in $k(x)$ and maximized in $k^\#(x)$ defined by (3.5). Comparing with (3.2) we see that $k^\#(x) > k^0(x)$. Thus,

$k^\#(x) > k^0(x) > k^*(x)$, implying that $k^0(x)$ and $k^*(x)$ lie in the increasing part of the function, thus $(p - \theta r - \tau(x - r)f(k^0(x))) - ik^0(x) > (p - \theta r - \tau(x - r)f(k^*(x))) - ik^*(x) \quad \forall x$. As trader's profit could always be increased, the case (2) cannot characterize the optimum. Eliminating case (2) from the possible outcomes, the first farmer's participation constraint can never be the only one to be binding at the equilibrium. \square

C Mill pricing: unconstrained outcome

The unconstrained outcome is the solution to the maximization problem when $\lambda = 0$ and $\mu = 0$. Plugging this in (6.3) and (6.4), and using $f(k) = 2\sqrt{k}$ gives us after simplification:

$$(p - \theta r - p_M) - \frac{i}{i_M} \left(p_M - \tau \frac{R}{2} \right) = 0 \quad (8.3)$$

$$(p - \theta r - p_M) \left(p_M - \tau \frac{R}{2} \right) + \left(\frac{1}{2} - \frac{i}{i_M} \right) \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right) = 0 \quad (8.4)$$

C.1 Characteristics of the unconstrained equilibrium

Equations (8.3) and (8.4) can be combined as $H(p_M) \equiv$

$$\left(p_M - \frac{\tau R}{2} \right) \left[\left(p_M - \frac{\tau R}{2} \right)^2 - \frac{\tau^2 R^2}{12} \right] + \frac{2\tau^2 R^2}{12} \left[2 \left(p_M - \frac{\tau R}{2} \right) - \left(p - \theta r - \frac{\tau R}{2} \right) \right] = 0$$

We have $H\left(\frac{\tau R}{2}\right) < 0$ and $H(p - \theta r) > 0$. In addition, we have $H'(p_M) > 0$ which means that there is a unique value for p_M between $\frac{\tau R}{2}$ and $p - \theta r$ such that $H(p_M) = 0$. If there is a solution such that $p_M > \tau R$, then $i_M < i$. To see this, note that whenever $p_M > \tau R$ the term between the first square brackets is positive which implies that the term between the second square brackets has to be negative. Plugging this in the equation (8.3) implies that $i_M < i$.

To establish under what conditions $p_M = p - \tau r$ we evaluate $H(p_M)$ at $p_M = p - \tau r$ which yields

$$n(p) \equiv H(p - \tau r) \equiv \left(p - \tau r - \frac{\tau R}{2} \right)^3 + \frac{\tau^2 R^2}{4} \left(p - \tau r - \frac{\tau R}{2} \right) - \frac{\tau^2 R^2}{6} \left(p - \theta r - \frac{\tau R}{2} \right)$$

We have

$$\begin{aligned}
\frac{dn(p)}{dp} &= 3 \left(p - \tau r - \frac{\tau R}{2} \right)^2 + \frac{\tau^2 R^2}{4} - \frac{\tau^2 R^2}{6} > 0 \\
n(0) &= \left(-\tau r - \frac{\tau R}{2} \right)^3 + \frac{\tau^2 R^2}{4} \left(-\tau r - \frac{\tau R}{2} \right) - \frac{\tau^2 R^2}{6} \left(-\theta r - \frac{\tau R}{2} \right) \\
&= - \left(\tau r + \frac{\tau R}{2} \right)^3 + \frac{\tau^2 R^2}{2} \left[-\frac{\tau R}{12} - \frac{1}{3} \left(\frac{3}{2} \tau r - \theta r \right) \right] < 0 \\
n(p_1) &= \left(\tau r - \theta r + \frac{\tau R}{2} \right)^3 + \frac{\tau^2 R^2}{4} \left(\tau r - \theta r + \frac{\tau R}{2} \right) - \frac{\tau^2 R^2}{6} \left(2\tau r - 2\theta r + \frac{\tau R}{2} \right) \\
&= \left(\tau r - \theta r + \frac{\tau R}{2} \right)^3 + \frac{\tau^2 R^2}{4} \left(\tau r - \theta r + \frac{\tau R}{2} \right) - \frac{\tau^2 R^2}{3} \left(\tau r - \theta r + \frac{\tau R}{4} \right) > 0
\end{aligned}$$

where $p_1 = 2\tau r - \theta r + \tau R$.

These three elements together imply that there is a unique $p_0 \in [0, p_1]$ such that $n(p_0) = 0$ and $p_M = p_0 - \tau r$.

C.2 Proof of $0 < dp_M/dp < 1$ if the optimum is unconstrained

Taking total derivatives of (8.3) and (8.4), setting them equal to zero and rearranging:

$$- \left(1 + \frac{i}{i_M} \right) \frac{dp_M}{dp} + \frac{i}{i_M^2} \left(p_M - \frac{\tau R}{2} \right) \frac{di_M}{dp} = -1 \quad (8.5)$$

$$\left(\frac{p - \theta r - p_M}{p_M - \frac{\tau R}{2}} - 2 \frac{i}{i_M} \right) \frac{dp_M}{dp} + \frac{\frac{i}{i_M^2} \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right)}{p_M - \frac{\tau R}{2}} \frac{di_M}{dp} = -1 \quad (8.6)$$

Using Cramer's rule, we can calculate dp_M/dp as:

$$\begin{aligned}
\frac{dp_M}{dp} &= \frac{-\frac{i}{i_M^2} \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right)}{p_M - \frac{\tau R}{2}} + \frac{i}{i_M^2} \left(p_M - \frac{\tau R}{2} \right)}{\left(1 + \frac{i}{i_M} \right) \frac{\frac{i}{i_M^2} \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right)}{p_M - \frac{\tau R}{2}} - \frac{i}{i_M^2} \left(p_M - \frac{\tau R}{2} \right) \left(\frac{p - \theta r - p_M}{p_M - \frac{\tau R}{2}} - 2 \frac{i}{i_M} \right)} \\
\Leftrightarrow \frac{dp_M}{dp} &= \frac{\frac{\tau^2 R^2}{12}}{\left(1 + \frac{i}{i_M} \right) \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right) + \left(p_M - \frac{\tau R}{2} \right)^2 \left(\frac{p - \theta r - p_M}{p_M - \frac{\tau R}{2}} - 2 \frac{i}{i_M} \right)}
\end{aligned}$$

From (8.3), $i/i_M = (p - \theta r - p_M)/(p_M - \tau R/2)$, thus:

$$\begin{aligned}
\Leftrightarrow \frac{dp_M}{dp} &= \frac{\frac{\tau^2 R^2}{12}}{\left(1 + \frac{i}{i_M} \right) \left(p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3} \right) + \left(p_M - \frac{\tau R}{2} \right)^2 \left(-\frac{i}{i_M} \right)} \\
&\Leftrightarrow \frac{dp_M}{dp} = \frac{1}{\frac{12}{\tau^2 R^2} \left(p_M - \frac{\tau R}{2} \right)^2 + 1 + \frac{i}{i_M}} \quad (8.7)
\end{aligned}$$

From this expression, $0 < dp_M/dp < 1$. □

C.3 Proof of $di_M/dp < 0$ if the optimum is unconstrained

Similarly, using Cramer's rule for the system of equations (8.5)-(8.6), we can calculate di_M/dp as:

$$\frac{di_M}{dp} = \frac{\left(1 + \frac{i}{i_M}\right) + \left(\frac{p - \theta r - p_M}{p_M - \frac{\tau R}{2}} - 2\frac{i}{i_M}\right)}{-\left(1 + \frac{i}{i_M}\right) \frac{\frac{i}{i_M^2} (p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3})}{p_M - \frac{\tau R}{2}} - \frac{i}{i_M^2} (p_M - \frac{\tau R}{2}) \left(\frac{p - \theta r - p_M}{p_M - \frac{\tau R}{2}} - 2\frac{i}{i_M}\right)}$$

From (8.3), $i/i_M = (p - \theta r - p_M)/(p_M - \tau R/2)$, thus:

$$\begin{aligned} \frac{di_M}{dp} &= \frac{p_M - \frac{\tau R}{2}}{-\left(1 + \frac{i}{i_M}\right) \frac{i}{i_M^2} (p_M^2 - p_M \tau R + \frac{\tau^2 R^2}{3}) - \frac{i}{i_M^2} (p_M - \frac{\tau R}{2})^2 \left(-\frac{i}{i_M}\right)} \\ &\Leftrightarrow \frac{di_M}{dp} = -\frac{p_M - \frac{\tau R}{2}}{\frac{i}{i_M^2} \left((p_M - \frac{\tau R}{2})^2 + \left(1 + \frac{i}{i_M}\right) \frac{\tau^2 R^2}{12} \right)} \end{aligned} \quad (8.8)$$

If $p_M > \tau R/2$ (which is verified if the constraints are satisfied), then $di_M/dp < 0$. \square

With the results that $p_M(p_0) = p_0 - \tau r$ and $dp_M/dp < 1$, we have that when $p > (<) p_0$ then $p_M < (>) p_0 - \tau r$. Another implication is that when $p < p_0$ then $p_M > p_0 - \tau r > \tau R$ which implies that $i > i_M$.

D Proof of Proposition 3

From Lemmas 1 and 2, we have three possible outcomes: at the equilibrium the participation constraint is binding (i) only for the last farmer, (ii) for all the farmers, or, (iii) for none of the farmers.

(i) If $g(r+R) = 0$ and $g(x) > 0$ for all $x \in [r, r+R[$, then this requires $\left. \frac{\partial g(x, p_M, i_M)}{\partial x} \right|_{x=r+R} < 0$ which implies by (8.2) that $i_M < i$.

(ii) If $g(x) = 0$ for all $x \in [r, r+R]$, then the stand-alone situation is replicated and we have that $i_M = i$.

(iii) If $g(x) > 0$ for all $x \in [r, r+R]$, then $p_M > \tau R$. From the previous section we know that then $i_M < i$.

If the outcome is constrained, then $p_M \leq p - \tau r < p - \theta r$. If the outcome is unconstrained, then using equation (8.3) it is easily seen that $p_M < p - \theta r$. \square

E Proof of Proposition 4

Based on (6.5) and (6.6), we identify four possible outcomes.

First possibility: $g(r+R) > 0$, $i_M < i$:

Let $(p_M^u(p), i_M^u(p))$ be the solution to the unconstrained problem given by the system of equations (8.3)-(8.4). Define $G(p) \equiv y(r+R, p) - y^0(r+R, p)$ where $y(r+R, p) =$

$y(r + R, p_M^u(p), i_M^u(p))$ if $p_M^u(p) \geq \tau R$ and $y(r + R, p) = 0$ if $p_M^u(p) < \tau R$. If $G(p) > 0$, then $(p_M^u(p), i_M^u(p))$ respects the constraint and given that $i_M < i$ as established in the proof of Proposition 3, the equilibrium is unconstrained. If $G(p) < 0$, however, then $(p_M^u(p), i_M^u(p))$ does not respect the constraint and the equilibrium has to be constrained.

In what follows, we establish that 1) when $\tau r - \theta r > \tau R$, $G(p)$ is positive for small values of p , i.e. for $p < p_0$, and when $\tau r - \theta r \leq \tau R$, $G(p)$ is negative for all values of p ; 2) $G(p)$ is negative for large values of p , i.e. for $p > p_1$; 3) $G(p)$ is strictly decreasing in p when $p_0 < p < p_1$.

Together this implies that when $\tau r - \theta r > \tau R$ for small values of p the equilibrium is unconstrained and that there exists a unique \bar{p} given by $G(\bar{p}) = 0$ above which the equilibrium is constrained. It also implies that when $\tau r - \theta r \leq \tau R$ the equilibrium is constrained for all values of p .

We proceed by establishing several intermediate results: (i) $p_M^u(p) < p - \tau r$ is a sufficient condition for $G(p)$ to be decreasing in p . (ii) $G(p) < 0$ for all $p > p_1$. (iii) There exists a unique p_0 , with $0 < p_0 < p_1$, such that $p_M^u(p_0) = p_0 - \tau r$; (iv) for $p \gtrless p_0$, $p_M^u(p) \gtrless p - \tau r$; (v) If $\tau r - \theta r \gtrless \tau R$, then $\underline{p} \gtrless p_0$.

(i) We show that if $p_M^u(p) < p - \tau r$, then $dG(p)/dp < 0$. Note that as $p > 0$ we have $\tau R < p - \tau r$. Suppose first that $\tau R \leq p_M^u(p) < p - \tau r$. Using (8.7) and

(8.8) we have that $\frac{dG(p)}{dp} = \frac{2(p_M^u(p) - \tau R)}{i} \frac{\left(\frac{i}{i_M^u(p)} + (p_M^u(p) - \tau R)(p_M^u(p) - \tau(R/2)) \frac{6}{\tau^2 R^2} \right)}{1 + \frac{i}{i_M^u(p)} + (p_M^u(p) - \tau(R/2))^2 \frac{12}{\tau^2 R^2}} - \frac{2(p - \tau r - \tau R)}{i}$. It

can be easily verified that the second ratio is smaller than 1, implying that $\frac{dG(p)}{dp} < \frac{2(p_M^u(p) - \tau R)}{i} - \frac{2(p - \tau r - \tau R)}{i}$. This is strictly negative provided that $p_M^u(p) < p - \tau r$. Suppose now that $p_M^u(p) < \tau R$, then $\frac{dG(p)}{dp} = -\frac{2(p - \tau r - \tau R)}{i}$, which is also negative.

(ii) Solving (8.3) for $i_M^u(p)$ and substituting it into $G(p)$ yields that $G(p) < 0$ if $\frac{(p_M^u(p) - \tau R)^2}{p_M^u(p) - \tau(R/2)} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p_M^u(p)}$. A sufficient condition for this to be verified is that:

$$\frac{(p_M^u(p) - \tau R)^2}{p_M^u(p) - \tau R} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p_M^u(p)}$$

$$\Leftrightarrow w(p_M^u(p)) \equiv -p_M^u(p)^2 + (p - \theta r + \tau R)p_M^u(p) - (p - \theta r)\tau R - (p - \tau r - \tau R)^2 < 0$$

$w(p_M^u(p))$ is a polynomial of degree two in $p_M^u(p)$ with a strictly negative leading coefficient. If the discriminant, given by $d(p) = (p - \theta r + \tau R)^2 - 4((p - \theta r)\tau R) - 4(p - \tau r - \tau R)^2$, is negative, then $w(p_M^u(p)) < 0$ for all $p_M^u(p)$, and hence $G(p) < 0$. We have that $d(p)$ is a polynomial (of degree two) in p with a strictly negative leading coefficient. As $\theta < \tau$, we have $p_1 \equiv 2\tau r + \tau R - \theta r > \underline{p}$. Note that $d(p_1) = 0$ and $d'(p_1) = -4r(\tau - \theta) < 0$. Thus $p > p_1$ is a sufficient condition for $w(p_M^u(p)) < 0$ for all $p_M^u(p)$. Hence $G(p) < 0$ for all $p > p_1$.

(iii) From Appendix C.2, there is a unique $p_0 \in [0, p_1]$ such that $p_M^u(p_0) = p_0 - \tau r$.

(iv) From Appendix C.2, $dp_M^u/dp < 1$, hence $p_M^u(p) \gtrless p - \tau r$ for all $p \gtrless p_0$. Substituting $p_M^u(p) \geq p - \tau r$ into $G(p)$, because $i_M^u(p_0) < i$, we have that $G(p) > 0$ for all $p \leq p_0$.

(v) The difference $p_0 - \underline{p}$ depends on the values of the parameters. To see this: $n(\underline{p}) = (\tau^2 R^2/6)(\tau R - (\tau r - \theta r))$. If $\tau r - \theta r > \tau R$, then $n(\underline{p}) < 0$ and $\underline{p} < p_0$. If $\tau r - \theta r \leq \tau R$, then $n(\underline{p}) > 0$ and $\underline{p} > p_0$. This implies that all (acceptable) values of $p > p_0$.

From (v) if $\tau r - \theta r \leq \tau R$, then all values of p are larger than p_0 . From (iv) this implies that $p_M^u(p) < p - \tau r$ for all values of p . This in turn implies that $p_M^u(\underline{p}) < \underline{p} - \tau r = \tau R$. From the definitions we have that $y(r+R, \underline{p}) = 0$ implying that $G(\underline{p}) < 0$. This, together with from (i), $dG(p)/dp < 0$ for $p > \underline{p}$, is sufficient to ensure that $G(p) < 0$ for all $p > \underline{p}$.

From (v) if $\tau r - \theta r > \tau R$, then there are values of $p \in [\underline{p}, p_0]$ such that $G(p) > 0$. For values of p larger than p_0 , $dG(p)/dp < 0$ and with values larger than p_1 , $G(p) < 0$ which implies that there is a unique \bar{p} such that $G(\bar{p}) = 0$.

Second possibility: $g(r+R) = 0$, $i_M < i$:

Let $(p_M^c(p), i_M^c(p))$ be the solution to the maximization problem when only the last farmer's participation constraint is binding, that is when $\mu = 0$ and $\lambda > 0$. Solving for λ in (6.4) and substituting it into (6.3), when $f(k) = 2\sqrt{k}$, gives:

$$i_M = i \frac{6p_M^2 - 3p_M\tau R + \tau^2 R^2}{6p_M(p - \tau R - \theta r) + 2\tau^2 R^2} \equiv i_M^\alpha(p_M) \quad (8.9)$$

The binding participation constraint $g(r+R) = 0$ gives:

$$i_M = i \frac{(p_M - \tau R)^2}{(p - \tau r - \tau R)^2} \quad (8.10)$$

Prices $(p_M^c(p), i_M^c(p))$ are given by the intersection between the curves (8.9) and (8.10), provided $\tau R \leq p_M^c(p) \leq p - \tau r$. Simplifying: $h(p_M^c(p)) \equiv$

$$(6p_M^{c2} - 3p_M^c\tau R + \tau^2 R^2)(p - \tau r - \tau R)^2 - (p_M^c - \tau R)^2(6p_M^c(p - \tau R - \theta r) + 2\tau^2 R^2) = 0$$

(a) $h(p_M^c(p))$ is a polynomial of degree three in $p_M^c(p)$ with a strictly negative leading coefficient. This implies that $h(p_M^c(p))$ has an inverse N-shape. (b) Evaluated at $p_M^c(p) = \tau R$, $h(\tau R) > 0$. (c) The first derivative of $h(p_M)$, evaluated at τR is strictly positive. This implies that τR lies in an increasing part of $h(p_M^c(p))$. (d) If $\tau r - \theta r > \tau R/2$ holds, then $h(p - \tau r) < 0$ for all $p > \underline{p}$. If $\tau r - \theta r \leq \tau R/2$ holds, then $h(p - \tau r) \leq 0$ holds for $p \leq \tilde{p}$ with $\tilde{p} = \tau r + \frac{\tau^2 R^2}{6(\tau(R/2) - \tau r + \theta r)}$.

Elements (a) to (d) are sufficient to ensure that if $\tau r - \theta r > \tau R/2$, then $h(p_M)$ has one unique root between τR and $p - \tau r$ for all $p > \underline{p}$. Hence, $\lambda > 0$ and $\mu = 0$ are possible for all the values of p we consider. If $\tau r - \theta r \leq \tau R/2$, then $h(p_M)$ has one unique root between τR and $p - \tau r$ when $p \leq \tilde{p}$ and no root between τR and $p - \tau r$ when $p > \tilde{p}$. Hence, $\lambda > 0$ and $\mu = 0$ only occur for $p \leq \tilde{p}$. Moreover, if $\tau r - \theta r < \tau R/3$, then $\tilde{p} < \underline{p}$ such that for all acceptable values of p , we have $p > \tilde{p}$.

Third possibility: $g(r+R) = 0$, $i_M = i$:

If $g(r + R) = 0$ and $i_M = i$, with $f(k) = 2\sqrt{k}$, then we have that $p_M = p - \tau r$. Replacing i_M by i and p_M by $p - \tau r$ in (6.3) and (6.4), solving for λ in (6.4) and substituting it into (6.3), we have:

$$\mu = -\frac{R}{i^2} \left((p - \tau r) \left(\tau r - \theta r - \tau \frac{R}{2} \right) + \frac{\tau^2 R^2}{6} \right) \quad (8.11)$$

If $\tau r - \theta r > \tau R/2$, then μ is always negative. If $\tau r - \theta r \leq \tau R/2$, then $\mu \leq 0$ if $p \leq \tilde{p}$ where $\tilde{p} = \tau r + \frac{\tau^2 R^2}{6(\tau(R/2) - \tau r + \theta r)}$ since $\mu = 0$ when $p = \tilde{p}$ and $\frac{\partial \mu}{\partial p} > 0$.

Fourth possibility: $g(r + R) > 0$, $i_M = i$:

If $g(r + R) > 0$, then from Appendix D (iii), we have that $i_M < i$. This implies that $g(r + R) > 0$ and $i_M = i$ never occurs.

Summarizing, this means that, if $\tau r - \theta r > \tau R$, then $g(r + R) > 0$ and $i_M < i$ for $p \in [\underline{p}, \bar{p}]$ while $g(r + R) = 0$ and $i_M < i$ for $p > \bar{p}$. If $\tau R/2 < \tau r - \theta r < \tau R$, then $g(r + R) = 0$ and $i_M < i$ for any $p > \underline{p}$. If $\tau r - \theta r < \tau R/2$, then $g(r + R) = 0$ and $i_M < i$ for $p \in [\underline{p}, \bar{p}]$ while $g(r + R) = 0$ and $i_M = i$ for $p > \bar{p}$. \square

F Parameter condition for complete market coverage

To establish the parameter conditions under which it is profitable for the trader to cover completely the market under all pricing policies, we only have to change a few elements to the above analysis. Now besides the two prices as choice variables, there is a third choice variable R_i (where $i = U, M$, or D). We replace in the Lagrangians R with R_i and add the term $\alpha (R - R_i)$ with $\alpha \geq 0$, $R - R_i \geq 0$ and $\alpha (R - R_i) = 0$. There is for each pricing policy a third condition which has to be verified. The two first order conditions remain the same (except for replacing R with R_i). For discriminatory pricing no additional restriction is required while the most restrictive condition is for uniform pricing. We show that under the condition for uniform pricing, the trader finds it optimal to cover completely the market under mill pricing.

For uniform pricing we have a third first order condition given by $\partial L / \partial R_U = 0$ or $((p - \theta r - \tau (R/2)) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)) - (\tau R/2) f(k^*) - \alpha = 0$. Using result (5.6) and the result that $y(r)^* = y^0(r)$, this can be written as $(p - \theta r - \tau (R/2))^2 - (p - \tau r)^2 - \tau R (p - \theta r - \tau (R/2)) = \alpha i$. It is easily verified that the LHS is positive when $p > p_{UR}(R) \equiv (\tau r - \theta r - \tau (R/2))^2 / 2(\tau r - \theta r - \tau R) + \theta r + \tau (R/2)$. We have that $p'_{UR}(R) > 0$ and $p_{UR}(0) < \underline{p}$. We also have that $p_{UR}(R) \rightarrow \infty$ as $R \rightarrow (\tau r - \theta r) / \tau$. For it to be profitable to cover the whole market under uniform pricing $p > \max [p_{UR}(R), \underline{p}]$.

For mill pricing the third first order condition given by $\partial\mathcal{L}/\partial R_M = 0$ or

$$\begin{aligned} & (p - \theta r - \tau R_M) f(k^*(r + R_M)) - ik^*(r + R_M) - y^*(r + R_M) \\ & + \lambda(-\tau f(k^*(r + R_M))) + \tau f(k^0(r + R_M)) - \alpha = 0 \\ \Leftrightarrow & (p - \theta r - \tau R_M) 2 \frac{(p_M - \tau R_M)}{i_M} - i \frac{(p_M - \tau R_M)^2}{i_M^2} - \frac{(p_M - \tau R_M)^2}{i_M} \\ & + 2\lambda\tau \left(\frac{(p - \tau r - \tau R_M)}{i} - \frac{(p_M - \tau R_M)}{i_M} \right) - \alpha = 0 \end{aligned}$$

There are three cases which have to be considered: the unconstrained case, the constrained case and the standalone case.

In the unconstrained case, $g(r + R_M) > 0$ which implies that $\lambda = 0$. Plugging this in the third first order condition and rearranging gives us $\partial\mathcal{L}/\partial R_M = 0$ or

$$\frac{(p_M - \tau R_M)}{i_M} \left\{ (p - \theta r - p_M) + (p_M - \tau R_M) + (p - \theta r - p_M) - i \frac{(p_M - \frac{\tau R_M}{2})}{i_M} + \frac{i}{i_M} \frac{\tau R_M}{2} \right\} - \alpha = 0$$

Using (8.3) and evaluating at $R_M = R$ we have that

$$\frac{(p_M - \tau R)}{i_M} \left\{ p - \theta r - \tau R + \frac{i}{i_M} \frac{\tau R_M}{2} \right\} = \alpha > 0$$

In the constrained case we have that $g(r + R_M) = 0$ which implies that $\lambda \geq 0$. Considering the case where $i > i_M$ and hence $\mu = 0$. It will be sufficient to show that the condition $(\tau - \theta)r > \tau R$ guarantees the market to be completely covered.

We can rewrite $\partial\mathcal{L}/\partial p_M = 0$ as $-(R/(p_M - \tau R)) \left((p - \theta r - p_M) - \frac{i}{i_M} (p_M - \frac{\tau R}{2}) \right) = \lambda$. Introducing this in the third FOC and rearranging the terms yields

$$\begin{aligned} \alpha i_M^2 &= -\frac{i_M^2 2\tau R (p - \tau r - \tau R)}{i (p_M - \tau R)} \left((p - \theta r - p_M) - \frac{i}{i_M} \left(p_M - \tau \frac{R}{2} \right) \right) \\ &+ i_M (2p_M (p - \theta r - p_M) + (p_M - \tau R)^2) - ip_M^2 \end{aligned}$$

This equation together with (8.9) and (8.10) gives us a system of three equations in three unknowns: p_M , i_M , and α . Introducing (8.10) in the last equation yields

$$\begin{aligned} \alpha i_M^2 &= -\frac{i_M 2\tau R (p_M - \tau R)}{(p - \tau r - \tau R)} \left((p - \theta r - p_M) - \frac{i}{i_M} \left(p_M - \tau \frac{R}{2} \right) \right) \\ &+ i_M (2p_M (p - \theta r - p_M) + (p_M - \tau R)^2) - ip_M^2 \end{aligned}$$

The LHS is linear and increasing in i_M . By setting $\alpha = 0$ in this equation we obtain the combinations of p_M and i_M such that $\alpha = 0$. This can be written as

$$i_M = i \frac{p_M^2 - \frac{2\tau R(p_M - \tau R)}{(p - \tau r - \tau R)} (p_M - \tau \frac{R}{2})}{2 \left(p_M - \frac{\tau R(p_M - \tau R)}{(p - \tau r - \tau R)} \right) (p - \theta r - p_M) + (p_M - \tau R)^2} \equiv i_M^\alpha (p_M)$$

We show that when $(\tau - \theta)r > \tau R$ we have that $i_M^\alpha(p_M) < i_M^c(p_M)$ for all $p_M \in [\tau R, p - \tau r]$. This implies that given that the solution lies on $i_M^c(p_M)$, this solution is characterised by $\alpha > 0$. To show this, note that we have that $i_M^c(p - \tau r) > i_M^\alpha(p - \tau r)$

$$i_M^\alpha(p - \tau r) = i \frac{p - \tau r - \tau R}{p - \tau r - \tau R + 2r(\tau - \theta)} < i \frac{(p - \tau r)^2 + (p - \tau r - \tau R/2)^2 + \frac{\tau^2 R^2}{12}}{2(p - \tau r)(p - \theta r - \tau R) + \frac{2\tau^2 R^2}{3}} = i_M^c(p - \tau r)$$

$$\Leftrightarrow D(p) \equiv -Ap^2 + Bp - C < 0$$

with $A = 6r(\tau - \theta) + 3\tau R$, $B = 2(Ar\tau + 2\tau^2 R^2)$, and $C = Ar^2\tau^2 + A\tau^2 R^2/3 + 4\tau^3 R^2 r$. To show that $D(p) < 0$, we note the following elements: $D(p)$ is quadratic in p , $D'' < 0$, $D(0) < 0$ and $D(p)$ has no roots. The discriminant of $D(p)$, given by $\Delta = B^2 - 4AC = 4\tau^2 R^2 (-12(\tau - \theta)^2 r^2 - 12R\tau(\tau - \theta)r + \tau^2 R^2) \equiv \Delta[(\tau - \theta)r]$, is negative because $\Delta'[(\tau - \theta)r] < 0$ since $\tau > \theta$ and $\Delta(\tau R/3) < 0$. Furthermore, when $(\tau - \theta)r > \tau R$ we have that $i_M^\alpha(\tau R) = i\tau R/2(p - \theta r - \tau R) < 2\tau Ri/3(p - \theta r) - 2\tau R = i_M^c(\tau R)$ since this is verified when $p > \theta r + 2\tau R$ which in this case is smaller than $\tau r + \tau R$. When $(\tau - \theta)r < \tau R < 3(\tau - \theta)r$, numerical simulations show that $\alpha > 0$ when $p > p^\alpha(R)$ with $p^\alpha((\tau - \theta)r/\tau)r = p^\alpha(3(\tau - \theta)r/\tau) = \underline{p}$ and $\min[\theta r + 2\tau R, \tilde{p}] > p^\alpha(R) > \underline{p}$. Finally, in the standalone case it is easily verified that $\alpha = 2r(\tau - \theta)(p - \tau(r + R))/i > 0$ since $\tau > \theta$.

For discriminatory pricing, using (4.1), (4.8) and the fact that at equilibrium $y^*(x) = y^0(x)$, we can write the profit at location x as $(\tau r - \theta r)(2p - \theta r - \tau(2x - r))/i$. Using this expression, we know that the profit is positive at $x = r + R$ if $p > \tau R + (\tau r + \theta r)/2$. This is verified since $\underline{p} > \tau R + (\tau r + \theta r)/2$ when $\theta < \tau$. □

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